A **B1A003** **Pages:3**

(2016 ADMISSIONS)

Reg. No:....................... Name:................................

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, FEBRUARY 2017

MA101: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer All Questions and each carries 5 marks)

1. a) Test the convergence of $\sum_{k=1}^{\infty}\frac{99^k}{k!}$ (2)

   b) Test the convergence of $\sum_{k=1}^{\infty}\frac{1}{3^k+1}$. (3)

2. a) Find the slope of the sphere $x^2+y^2+z^2=1$ in the y- direction at $(\frac{2}{3},\frac{1}{3},\frac{-2}{3})$. (2)

   b) Find the critical points of the function $f(x,y) = 2xy-x^3-y^3$. (3)

3. a) Find the velocity at time $t=\pi$ of a particle moving along the curve

   $\vec{r}(t) = e^t \sin t\, i + e^t \cos t\, j + t\, k$. (2)

   b) Find the directional derivative of $f(x,y) = xe^y-ye^x$ at the point P(0,0) in the direction of $5i - 2j$. (3)

4. a) Change the order of integration in $\int_0^1\int_{y^2}^{\sqrt{2-y^2}} f(x,y)dxdy$. (3)

   b) Find the area of the region enclosed by $y= x^2$ and $y = x$. (2)

5. a) Find the divergence of the vector field $f(x,y,z) = x^2y\, i + 2y^3z\, j + 3z\, k$. (2)

   b) Find the work done by $\overrightarrow{F} = xy\, i + x^3 j$ on a particle that moves along the

   curve $y^2=x$ from (0,0) to (0,1). (3)

6. a) Using Green's theorem to evaluate $\int_C 2xy \quad dx \quad + (x^2 + x)\, dy$ where C is the triangle with vertices (0,0), (1,0) and (1,1). (2)

   b) Use Stoke's theorem to evaluate $\int_C \vec{F}. \mathrm{dr}$ where $\vec{F} = (x-2y)i + (y-z)\, j + (z-x)\, k$ and C is the circle $x^2+y^2 = a^2$ in the xy plane with counter clockwise orientation looking down the positive z- axis. (3)

PART B

MODULE I *(Answer Any Two Questions)*

7. a) Test the convergence of the following series (5)

   i) $\sum_{k=1}^{\infty} \frac{(k+4)!}{4!k!\,4^k}$ ii) $\sum_{k=2}^{\infty} \left(\frac{4k-5}{2k+1}\right)^k$

8. Use the alternating series test to show that the series $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{(k+3)}{k(k+1)}$ converge. (5)

9. Find the Taylor's series of f(x) = x sinx about the point x= $\frac{\pi}{2}$. (5)

MODULE II *(Answer Any Two Questions)*

10. Find the local linear approximation L to f(x,y) = ln (xy) at P(1,2) and compare the error in approximating f by L at Q(1.01, 2.01) with the distance between P and Q. (5)

11. Show that the function f (x,y) = $2\tan^{-1}(y/x)$ satisfies the Laplace's equation

    $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$ (5)

12. Find the relative minima of f(x,y) = $3x^2-2xy+y^2-8y$. (5)

MODULE III *(Answer Any Two Questions)*

13. Find the unit tangent vector and unit normal vector to $\vec{r} = 4\cos t\,i + 4\sin t\,j + t\,k$ at t = $\frac{\pi}{2}$. (5)

14. Suppose a particle moves through 3- space so that its position vector at time t is $\vec{r} = t\,i + t^2 j + t^3 k$. Find the scalar tangential component of acceleration at the time t=1. (5)

15. Given that the directional derivative of f(x,y,z) at (3,-2, 1) in the direction of $2i - j - 2k$ is -5 and that $\|\nabla f(3,-2,1)\| = 5$. Find $\nabla f(3,-2,1)$. (5)

MODULE IV *(Answer Any Two Questions)*

16. Evaluate the integral $\int_0^4 \int_{\sqrt{y}}^2 e^{x^3}\, dxdy$ by reversing the order of integration. (5)

17. Evaluate $\int_0^1 \int_0^{y^2} \int_{-1}^{z} z\, dx\, dy.$ (5)

18. Find the volume of the solid in the first octant bounded by the co-ordinate planes and the plane x+2y+z = 6. (5)

MODULE V ***(Answer Any Three Questions)***

19. Let $\vec{r}= xi + yj + zk$ and let $r = \|\vec{r}\|$ and f be a differentiable function of one variable show that $\nabla f(r) = \frac{f'(r)}{r}\vec{r}$. (5)

20. Evaluate the line integral $\int_C [-y\, dx + x\, dy]$ along $y^2 = 3x$ from (3,3) to (0,0). (5)

21. Show that $\overrightarrow{F}(x,y) = (\cos y + y \cos x)\, i + (\sin x - x \sin y)\, j$ is a conservative vector field. Hence find a potential function for it. (5)

22. Show that the integral $\int_C (3x^2 e^y\, dx + x^3 e^y\, dy)$ is independent of the path and hence evaluate the integral from (0,0) to (3,2). (5)

23. Find the work done by the force field $\overrightarrow{F} = xy\, i + yz\, j + xz\, k$ on a particle that moves along the curve C: $\vec{r}(t) = t\, i + t^2\, j + t^3\, k$ where $0 \le t \le 1$. (5)

MODULE VI ***(Answer Any Three Questions)***

24. Use Green's theorem to evaluate the integral $\int_C (x \cos y\, dx - y \sin x\, dy)$ where C is the square with vertices (0,0), $(\pi,0)$, (π,π) and $(0,\pi)$. (5)

25. Evaluate the surface integral $\int_\sigma \int z^2\, ds$ where σ is the portion of the cone $z = \sqrt{x^2 + y^2}$ between the planes z = 1 and z = 3. (5)

26. Use divergence theorem to find the outward flux of the vector field $\overrightarrow{F} = 2x\, i + 3y\, j + z^2\, k$ across the unit cube x = 0, x = 1, y=0, y = 1, z = 0 and z = 1. (5)

27. Use Stoke's theorem to evaluate the integral $\int_C \overrightarrow{F} . d\vec{r}$ where $\overrightarrow{F} = (x-y)\, i + (y-z)\, j + (z-x)k$ and C is the boundary of the portion of the plane x+y+z = 1 in the first octant. (5)

28. Use Stoke's theorem to evaluate the integral $\int_C \overrightarrow{F} . d\vec{r}$ where $\overrightarrow{F} = 2z\, i + 3x\, j + 5y\, k$ and C is the boundary of the paraboloid $x^2+y^2+z = 4$ for which $z \ge 0$ and C is positively oriented. (5)

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