A   **B1A216S (2015 Admission)**   **Pages:2**

Reg.No......................   Name..............................

Max.Marks : 100   Duration : 3 Hours

PART A

(Answer all questions. Each question carries 3 marks)

1) Show that the series $\sum_{n=1}^{\alpha} \left(\frac{1}{2}\right)^n$ converges.

2) Classify the surface $z = (x-1)^2 + (y+2)^2 + 3$

3) Find the Maclaurin series for $cos\ x$

4) Evaluate $\underset{(x,y)\to(-1,2)}{Lt} \dfrac{xy}{x^2 + y^2}$

5) Convert the cylindrical co-ordinate into rectangular co ordinate of $(4, \pi/3 - 3)$.

6) Find the slope of the surface $z = xy^2$ in the x direction at the point (2,3).

7) Find the directional derivative of $f = x^2 y - y z^3 + z$ at (1,-2,0) in the direction of

$\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$

8) Find the unit normal to the surface $xy + xz + yz = c$ at (-1,2,3)

9) Evaluate $\int_1^a \int_1^b x^2 y\,dxdy$

10) Find the area of the region R enclosed by $y = 1, y = 2, x = 0, x = y$.

PART B

(Answer any 2 questions. Each question carries 7 marks)

11) Test the absolute convergence of $\sum_{n=1}^{\alpha} \dfrac{(-1)^n n^4}{4^n}$

12) Determine the Taylor's series expansion of f(x) = $sin\ x$ at x = π/2.

13) Test the convergence of $\dfrac{1}{1.2.3} + \dfrac{3}{2.3.4} + \dfrac{5}{3.4.5}$ ....

(Answer any 2 questions. Each question carries 7 marks)

14) Find the equation of the paraboloid $z = x^2 + y^2$ in the cylindrical and spherical coordinates.

15) Find $F(f(x),g(y),h(z))$ if $F(x,y,z)=y\,e^{xyz}$, $f(x)=x^2$, $g(y)=y+1$, $h(z)=2z^2$

16) By converting into polar coordinate evaluate $\underset{(x,y)\to(0,0)}{Lt}\ \sqrt{x^2+y^2}\ \ln\left(\left(x^2+y^2\right)^2\right)$

(Answer any 2 questions. Each question carries 7 marks)

17) Find the local linear approximation L of $f(x,y,z) = xyz$ at the point P(1,2,3). Compare the error in approximating f by L at the point Q(1.001, 2.002, 3.003) with the distance PQ.

18) Find the relative extrema of $f(x,y) = 3x^2 - 2xy + y^2 - 8y$

19) If f is a differentiable function of three variables and suppose that

$$w = f(x-y, y-z, z-x) \quad \text{Show that } \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$$

(Answer any 2 questions. Each question carries 7 marks)

20) Suppose that a particle moves along a curve in 3-space so that its position vector at time t is $r(t) = 4\cos \pi t\, i + 4\sin \pi t\, j + t\, k$. Find the distance travelled and the displacement of the particle during the time interval $1 \le t \le 5$

21) A particle is moving along the curve, $\vec{r} = (t^3 - 2t)\vec{i} + (t^2 - 4)\vec{j}$ where t denotes the time. Find the scalar tangential and normal components of acceleration at t = 1. Also find the vector tangential and normal components of acceleration at t = 0.

22) Find the arc length of the parametric curve $x = 5\cos t,\ y = 5\sin t,\ z = 2t\ ;\ 0 \le t \le \pi$

(Answer any 2 questions. Each question carries 7 marks)

23) Evaluate the integral by converting into polar co ordinates $\int\limits_{0}^{2}\int\limits_{0}^{\sqrt{4-x^2}} (x^2 + y^2)\,dy\,dx$

24) Using triple integral to find the volume bounded by the cylinder

$$x^2 + y^2 = 4\ and\quad the\ planes\ z = 0\ and\ y + z = 3$$

25) Change the order of integration and evaluate $\int\limits_{0}^{1}\int\limits_{x}^{1} \frac{x}{x^2 + y^2}\,dx\,dy$

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