

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
SECOND SEMESTER B.TECH DEGREE EXAMINATION, JULY 2017

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions. Each carries 3 marks.

- 1 Find a second order homogeneous linear ODE for which e^{-x} and e^{-2x} are the solutions. (3)
- 2 Find a basis of solutions of $y^{11} - y^1 = 0$. (3)
- 3 Find the particular integral of $(D^2 - 4)y = x^2$. (3)
- 4 Solve $(D^2 + 3D + 2)y = 5$. (3)
- 5 Expand $\pi x - x^2$ in a half range sine series in the interval $(0, \pi)$. (3)
- 6 Expand $f(x)$ in Fourier series in the interval $(-2, 2)$ when

$$f(x) = \begin{cases} 0 & -2 < x < 0 \\ 1 & 0 < x < 2 \end{cases}$$
 (3)
- 7 Obtain the partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2)$. (3)
- 8 Solve $xp + yq = 3z$. (3)
- 9 Using the method of separation of variables solve $u_{xy} - u = 0$. (3)
- 10 Write down the possible solutions of one dimensional wave equation. (3)
- 11 Find the solution of one dimensional heat equation in steady state condition. (3)
- 12 State one dimensional heat equation with boundary conditions and initial conditions for solving it. (3)

PART B

Answer six questions, one full question from each module.

Module 1

- 13 a) Reduce to first order and solve $x^2y^{11} - 5xy^1 + 9y = 0$. Given $y_1 = x^3$ is a solution. (6)
- b) Solve the initial value problem $4y^{11} - 25y = 0$ where $y(0) = 0$, $y^1(0) = -5$. (5)

OR

- 14 a) Show that the functions $e^{-x}\text{Cos}x$ and $e^{-x}\text{Sin}x$ are linearly independent. Form a second order linear ODE having these functions as solutions. (6)
- b) Solve $y^{1V} - 2y^{111} + 5y^{11} - 8y^1 + 4y = 0$. (5)

Module 1I

- 15 a) Solve $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10 \left(x + \frac{1}{x} \right)$. (6)
- b) Solve $y^{11} - 4y^1 + 3y = e^x \text{Cos} 2x$. (5)

OR

- 16 a) Solve $y^{11} + y = \text{Cosec} x$ using the method of variation of parameters. (6)
- b) Solve $(D^2 - 2D + 1)y = x \text{Sin}x$. (5)

Module III

- 17 a) If $f(x) = x + x^2$ for $-\pi < x < \pi$ find the Fourier series expansion of $f(x)$. (6)
 b) Express $f(x) = |x|$ $-\pi < x < \pi$ as Fourier series. (5)

OR

- 18 a) Obtain Fourier series for the function $f(x) = \begin{cases} \pi x & \text{when } 0 \leq x \leq 1 \\ \pi(2-x) & \text{when } 1 \leq x \leq 2 \end{cases}$ (6)
 b) Obtain the half range cosine series for $f(x) = x$ in the 2interval $0 \leq x \leq \pi$. (5)
 Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$.

Module IV

- 19 a) Solve $xp - yq = y^2 - x^2$. (6)
 b) Solve $\frac{\partial^2 z}{\partial x^2} - 7 \frac{\partial^2 z}{\partial x \partial y} + 12 \frac{\partial^2 z}{\partial y^2} = e^{x-y}$. (5)

OR

- 20 a) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \text{Sin}x \text{Cos}2y$. (6)
 b) Solve $p - 2q = 3x^2 \text{Sin}(y + 2x)$. (5)

Module V

- 21 Derive one dimensional wave equation. (10)

OR

- 22 a) A tightly stretched homogeneous string of length l with its fixed ends at $x = 0$ and $x = l$ executes transverse vibrations. Motion starts with zero initial velocity by displacing the string into the form $f(x) = k(x^2 - x^3)$. Find the deflection $u(x, t)$ at any time t . (10)

Module VI

- 23 Find the temperature distribution in a rod of length $2m$ whose end points are maintained at temperature zero and the initial temperature is $f(x) = 100(2x - x^2)$. (10)

OR

- 24 A long iron rod with insulated lateral surface has its left end maintained at a temperature 0°C and its right end at $x=2$ maintained at 100°C . Determine the temperature as a function of x and t if the initial temperature is (10)

$$u(x, 0) = \begin{cases} 100x & 0 < x < 1 \\ 100 & 1 < x < 2 \end{cases}$$
