

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**SECOND SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018**

**Course Code: MA102**

**Course Name: DIFFERENTIAL EQUATIONS**

Max. Marks: 100

Duration: 3 Hours

**PART A**

*Answer all questions, each carries 3 marks.*

- |    |   | Marks |
|----|---|-------|
| 1  | Solve the initial value problem $xy' = y - 1, y(0) = 1$   | (3)   |
| 2  | Solve the following differential equation by reducing it to first order $xy'' = 2y'$  | (3)   |
| 3  | Find the particular integral of $(D^2 + 3D + 2)y = 3$   | (3)   |
| 4  | Find the particular integral of $y'' + y = \sin x$  | (3)   |
| 5  | Obtain the Fourier series expansion for the function $f(x) = x$ in the range $-\pi < x < \pi$   | (3)   |
| 6  | Find the Fourier sine series of the function $f(x) = \pi x - x^2$ in the interval $(0, \pi)$  | (3)   |
| 7  | Form a partial differential equation by eliminating the arbitrary function in $xyz = \phi(x + y + z)$                                       | (3)   |
| 8  | Solve $r + s - 2t = e^{x+y}$  | (3)   |
| 9  | Solve one dimensional wave equation for $k < 0$   | (3)   |
| 10 | Solve $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} - u = 0, u(x, 0) = 6e^{-3x}$ using method of separation of variables. | (3)   |
| 11 | Find the steady state temperature distribution in a rod of length 30cm if the ends are kept at $20^\circ\text{C}$ and $80^\circ\text{C}$ .  | (3)   |
| 12 | Write down the possible solutions of one dimensional heat equation.   | (3)   |

**PART B**

*Answer six questions, one full question from each module.*

**Module I**

- 13 a) Verify that the given functions  $x^{\frac{3}{2}}, x^{-\frac{1}{2}}$  are linearly independent and form a basis of solution space of given ODE  $4x^2y'' - 3y = 0$ . (6)
- b) Solve the boundary value problem: (5)
- $$y'' - 10y' + 25y = 0, \quad y(0) = 1, \quad y(1) = 0.$$

**OR**

- 14 a) Find the general solution of  $y'''' + 2y'' + y = 0$ . (6)
- b) Find a fundamental set of solutions of  $2t^2y'' + 3ty' - y = 0, t < 0$ . Given that  $y_1(t) = \frac{1}{t}$  is a solution. (5)

**Module II**

- 15 a) Find the particular integral of  $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4\cos^2x$ . (6)
- b) Solve  $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$  using method of variation of parameters. (5)

**OR**

- 16 a) Solve  $x^2y'' - xy' - 3y = x^2 \ln x$  (6)
- b) Solve  $y'''' - 2y''' + 5y'' - 8y' + 4y = e^x$ . (5)

**Module III**

- 17 a) Obtain the Fourier series expansion of  $f(x) = x \sin x$  in the interval  $(-\pi, \pi)$ . (6)  
 b) Find the half range sine series of  $f(x) = k$  in the interval  $(0, \pi)$ . (5)

**OR**

- 18 a) Find the Fourier series of  $f(x) = \left(\frac{\pi-x}{2}\right)^2$  in the interval  $(0, 2\pi)$ . (6)  
 b) Find the half range sine series of  $f(x) = e^x$  in  $(0, 1)$ . (5)

**Module IV**

- 19 a) Solve  $\frac{\partial^3 z}{\partial x^3} - 4 \frac{\partial^3 z}{\partial x^2 \partial y} + 4 \frac{\partial^3 z}{\partial x \partial y^2} = 2 \sin(3x + 2y)$  (6)  
 b) Solve  $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$  (5)

**OR**

- 20 a) Form the PDE by eliminating  $a, b, c$  from  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  (6)  
 b) Solve  $(x + y)zp + (x - y)zq = x^2 + y^2$ . (5)

**Module V**

- 21 A tightly stretched violin string of length 'a' and fixed at both ends is plucked at its mid-point and assumes initially the shape of a triangle of height 'h'. Find the displacement  $u(x, t)$  at any distance 'x' and any time 't' after the string is released from rest. (10)

**OR**

- 22 Solve the PDE  $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ . (10)  
 Boundary conditions are  $u(0, t) = u(l, t) = 0, t \geq 0$   
 Initial conditions are  $y(x, 0) = a \sin\left(\frac{\pi x}{l}\right)$  and  $\frac{\partial y}{\partial t} = 0$  at  $t = 0$ .

**Module VI**

- 23 A rod, 30 cm long has its ends A and B kept at  $20^\circ\text{C}$  and  $80^\circ\text{C}$  respectively, until the steady state conditions prevail. The temperature at each end is then suddenly reduced to  $0^\circ\text{C}$  and kept so. Find the resulting temperature function  $u(x, t)$  taking  $x=0$  at A. (10)

**OR**

- 24 A long iron rod with insulated lateral surface has its left end maintained at a temperature  $0^\circ\text{C}$  and its right end at  $x=2$ , maintained at  $100^\circ\text{C}$ . Determine the temperature as a function of 'x' and 't' if the initial temperature is  
 $u(x, 0) = \begin{cases} 100x, & 0 < x < 1 \\ 100, & 1 < x < 2 \end{cases}$  (10)

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