Reg No.:_______________ Name:___________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER B.TECH DEGREE EXAMINATION, JULY 2018

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks

Marks

1 Consider the initial value problem $y'' - x^3 y' + 6xy = \sin x,\ y(0)=3, y'(0)=-1$. Can this problem have unique solution in an interval containing zero? Explain. (3)

2 Find any three independent solutions of the differential equation $y''' - y' = 0$. (3)

3 Find the particular solution of the differential equation $y'' - 6y' + 9y = e^{3x}$. (3)

4 Using a suitable transformation, convert the differential equation $(2x-3)^2 y'' - (2x-3)y' + 2y = (2x-3)^2$ into a linear differential equation with constant coefficients. (3)

5 State the conditions for which a function f(x) can be represented as a Fourier series. (3)

6 Discuss the convergence of a Fourier series of a periodic function f(x) of period 2π. (3)

7 Find the partial differential equation representing the family of spheres whose centers lies on z-axis. (3)

8 Find the particular solution of $(D^2 - 2DD' + 2D'^2)z = \sin(x-y)$ (3)

9 Write any three assumptions involved in the derivation of one dimensional wave equation. (3)

10 A string of length l fastened at both ends. The midpoint of the string is taken to a height h and then released from rest in that position. Write the boundary conditions and initial conditions of the string to find the displacement function y(x,t) satisfying the one dimensional wave equation. (3)

11 Write the fundamental postulates used in the derivation of one dimensional heat equation. (3)

12 Define steady state condition in one dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$. (3)

PART B

Answer six questions,one full question from each module

Module 1

13 a) Discuss the existence and uniqueness of solution of the initial value problem (6)

$$\frac{dy}{dx} = \frac{y}{\sqrt{x}},\ y(1)=3.$$

b) Prove that $y_1(x) = e^x$ and $y_2(x) = e^{4x}$ form a fundamental system(basis) for the (5) differential equation $y'' - 5y' + 4y = 0$. Can $5e^{x} - 2e^{4x}$ be a solution(do not use verification method) of the differential equation? Explain.

OR

14 a) Discuss the existence and uniqueness of solution of the initial value problem (6)

$$\frac{dy}{dx} = x^2 + y^2 \text{ , } y(0)=1 \text{ in the rectangle } |x| \le 1, |y-1| \le 1.$$

b) If $y_1(x)=x$ is a solution of $x^2 y'' + 2xy' - 2y = 0$, find the general solution. (5)

Module II

15 a) By the method of variation of parameters, solve $y'' + y = x\sin x$. (6)

b) Solve $y'' + 5y' + 6y = e^{-2x}\sin 2x$. (5)

OR

16 a) Solve $x^2 y'' + xy' - 9y = \log x$. (6)

b) Solve $y'' - 2y' + 5y = x^2$. (5)

Module III

17 Find the Fourier cosine series representation of $f(x)=x, 0 \le x \le \pi$. Also find the Fourier series representation f(x) if f(x) is periodic function with period π. (11)

OR

18 Find the Fourier series of the periodic function f(x) of period 4, where $f(x)=\begin{cases} 2, & -2 < x \le 0 \\ x, & 0 < x < 2 \end{cases}$ and deduce that (11)

$$(i)\ 1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+....=\frac{\pi^2}{8} \text{ and } (ii)\ 1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+....=\frac{\pi}{4}$$

Module IV

19 a) Find the particular solution of $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = y^2$. (5)

b) Find the general solution of $(y^2 + z^2)p - xyq = -xz$. (6)

OR

20 a) Solve $(D^2 + 3DD' + 2D'^2)z = (2x+y)^7$. (5)

b) Solve $4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 16\log(x+2y)$. (6)

Module V

21 a) Using method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} - u$, $u(x,0)=5e^{-3x}$. (5)

b) A tightly stretched string of length *l* fastened at both ends is initially in a position given by y = kx, 0 < x < l. If it is released from rest from this position, find the displacement y(x,t) at any time t and any distance x from the end x = 0. (5)

OR

22 A string is stretched and fastened in two points 50 cm apart. Motion is started by (10)

displacing the string into the form of the curve y = x(50 - x) and also by imparting a constant velocity V to every point of the string in the position at time t = 0. Determine the displacement function y(x, t).

Module VI

23 A rod of length 50 cm has its ends A and B kept at 20^0C and 70^0Crespectively until steady state temperature prevail. The temperature at each end is thensuddenly reduced to zero temperature and kept so. Find the resulting temperature function u(x,t) taking x = 0 at A. (10)

OR

24 A bar 10 cm long with insulated sides has its ends A and B maintainedat 50^0C and 100^0C respectively until steady state conditions prevail. The temperature atA is suddenly raised to 90^0C and at the same time that at B is lowered to 60^0C. Find thetemperature distribution in the bar at time t. (10)

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