

Reg. No. \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**THIRD SEMESTER B.TECH DEGREE EXAMINATION, MARCH 2017**

**MA 201: LINEAR ALGEBRA AND COMPLEX ANALYSIS**

Max. Marks: 100

Duration: 3 Hours

**PART A**

*Answer any 2 questions*

1. a. Check whether the following functions are analytic or not. Justify your answer.
  - i)  $f(z) = z + \bar{z}$  (4)
  - ii)  $f(z) = |z|^2$  (4)
- b. Show that  $f(z) = \sin z$  is analytic for all  $z$ . Find  $f'(z)$  (7)
2. a. Show that  $v = 3x^2y - y^3$  is harmonic and find the corresponding analytic function
 
$$f(z) = u(x, y) + iv(x, y)$$
 (8)
  - b. Find the image of  $0 < x < 1, \frac{1}{2} < y < 1$  under the mapping  $w = e^z$  (7)
3. a. Find the linear fractional transformation that carries  $z_1 = -2, z_2 = 0$  and  $z_3 = 2$  on to the points  $w_1 = \infty, w_2 = 1/4$  and  $w_3 = 3/8$ . Hence find the image of x-axis. (7)
  - b. Find the image of the rectangular region  $-\pi \leq x \leq \pi, a \leq y < b$  under the mapping  $w = \sin z$  (8)

**PART B**

*Answer any 2 questions*

4. a. Evaluate  $\int_C |z| dz$  where
  - i)  $C$  is the line segment joining  $-i$  and  $i$  (3)
  - ii)  $C$  is the unit circle in the left of half plane (4)
- b. Verify Cauchy's integral theorem for  $z^2$  taken over the boundary of the rectangle with vertices  $-1, 1, 1+i, -1+i$  in the counter clockwise sense. (8)
5. a. Find the Laurent's series expansion of  $f(z) = \frac{1}{1-z^2}$  which is convergent in
  - i)  $|z - 1| < 2$  (4)
  - ii)  $|z - 1| > 2$  (4)
- b. Determine the nature and type of singularities of
  - i)  $\frac{e^{-z^2}}{z^2}$  (3)

ii)  $z \sin\left(\frac{1}{z}\right)$  (4)

6. a. Use residue theorem to evaluate  $\int_C \frac{30z^2 - 23z + 5}{(2z - 1)^2(3z - 1)} dz$  where C is  $|z| = 1$  (7)

b. Evaluate  $\int_0^\infty \frac{1}{(1 + x^2)^2} dx$  using residue theorem. (8)

**PART C**

*Answer any 2 questions*

7. a. Solve the following by Gauss elimination

$$y + z - 2w = 0, \quad 2x - 3y - 3z + 6w = 2, \quad 4x + y + z - 2w = 4 \quad (6)$$

b. Reduce to Echelon form and hence find the rank of the matrix

$$\begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix} \quad (6)$$

c. Find a basis for the null space of  $\begin{bmatrix} 2 & -2 & 0 \\ 0 & 4 & 8 \\ 2 & 0 & 4 \end{bmatrix}$  (8)

8. a. i) Are the vectors  $(3 \ -1 \ 4)$ ,  $(6 \ 7 \ 5)$  and  $(9 \ 6 \ 9)$  linearly dependent or independent? Justify your answer. (5)

ii) Is all vectors  $(x, y, z)$  in  $\mathbb{R}^3$  with  $y - x + 4z = 0$  form a vector space over the field of real numbers? Give reasons for your answer. (5)

b. i) Find a matrix **C** such that  $Q = \mathbf{x}^T \mathbf{C} \mathbf{x}$  where

$$Q = -3x_1^2 + 4x_1x_2 - x_2^2 + 2x_1x_3 - 5x_3^2 \quad (4)$$

ii) Obtain the matrix of transformation

$$y_1 = \cos \theta x_1 - \sin \theta x_2, \quad y_2 = \sin \theta x_1 + \cos \theta x_2$$

Prove that it is orthogonal. Obtain the inverse transformation. (6)

9. a. Find the eigenvalues, eigenvectors and bases and dimensions for each Eigen space of

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \quad (10)$$

b. Find out what type of conic section, the quadratic form  $17x_1^2 - 30x_1x_2 + 17x_2^2 = 128$  and transform it to principal axes. (10)