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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: MA201

Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer any two full questions, each carries 15 marks Marks

1 a) Let $f(z) = u(x,y) + iv(x,y)$ be defined and continuous in some neighbourhood of a point $z = x + iy$ and differentiable at z itself. Then prove that the first order partial derivatives of u and v exist and satisfy the Cauchy – Riemann equations. (7)

b) Prove that $u = \sin x \cosh y$ is harmonic. Hence find its harmonic conjugate. (8)

2 a) Find the image of the region $\left|z - \frac{1}{3}\right| \leq \frac{1}{3}$ under the transformation $w = \frac{1}{z}$ (8)

b) Find a linear fractional transformation which maps $-1, 0, 1$ onto $1, 1+i, 1+2i$. (7)

3 a) Check whether the function $f(z) = \begin{cases} \frac{Re\,(z^2)}{|z|^2} & if\ z \neq 0 \\ 0 & if\ z = 0 \end{cases}$ is continuous at $z = 0$. (7)

b) Find the image of the x-axis under the linear fractional transformation $w = \frac{z+1}{2z+4}$ (8)

PART B

Answer any two full questions, each carries 15 marks

4 a) Evaluate $\int_C Im(z^2)dz$ where C is the triangle with vertices $0, 1, i$ counter-clockwise. (7)

b) Using Cauchy's Integral Formula, evaluate $\int_c \frac{z^2}{z^3 - z^2 - z + 1}\, dz$ where c is taken counter-clockwise around the circle: (8)

i) $|z+1| = \frac{3}{2}$ ii) $|z - 1 - i| = \frac{\pi}{2}$

5 a) Determine and classify the singular points for the following functions: (7)

i) $f(z) = \frac{\sin z}{(z-\pi)^2}$ ii) $g(z) = (z+i)^2 e^{\left(\frac{1}{z+i}\right)}$

b) Evaluate $\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^3}\, dx$. (8)

6 a) Evaluate $\int_C \frac{\tan z}{z^2 - 1}\, dz$ counter clockwise around c: $|z| = \frac{3}{2}$ using Cauchy's Residue Theorem. (7)

b) Find all Taylor series and Laurent series of $f(z) = \frac{-2z+3}{z^2 - 3z + 2}$ with centre 0 in (8)

i) $|z| < 1$ ii) $1 < |z| < 2$.

PART C

Answer any two full questions, each carries 20 marks

7 a) Solve the system of equations by Gauss Elimination Method: (8)
$3x+3y+2z=1, \quad x+2y=4, \quad 10y+3z=-2, \quad 2x-3y-z=5.$

b) Prove that the vectors $(1,1,2),(1,2,5),(5,3,4)$ are linearly dependent. (6)

c) Prove that the set of vectors $V=\{(v_1,v_2,v_3)\in\mathbb{R}^3 : -v_1+v_2+4v_3=0\}$ a vector space over the field $\mathbb{R}$. Also find the dimension and the basis. (6)

8 a) Find the Eigen values and the corresponding Eigen vectors of $A=\begin{bmatrix}1 & 1 & -2\\ -1 & 2 & 1\\ 0 & 1 & -1\end{bmatrix}$ (8)

b) What kind of conic section is given by the quadratic form $7{x_1}^2+6x_1x_2+7{x_2}^2=200.$ Also find its equation. (6)

c) Determine whether the matrix $A=\begin{bmatrix}1 & 0 & 0\\ 0 & cos\theta & -sin\theta\\ 0 & sin\theta & cos\theta\end{bmatrix}$ symmetric, skew-symmetric or orthogonal. (6)

9 a) Reduce the matrix $A=\begin{bmatrix}2 & 3 & -1 & -1\\ 1 & -1 & -2 & -4\\ 3 & 1 & 3 & -2\\ 6 & 3 & 0 & -7\end{bmatrix}$ to Row Echelon Form and hence find its rank. (8)

b) Diagonalize $A=\begin{bmatrix}3 & -1 & 1\\ -1 & 3 & -1\\ 1 & -1 & 3\end{bmatrix}$ (12)

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