

Reg. No. _____ Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION, DEC 2016

Course Code: MA201

Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100

Duration:3. Hours

PART A

(Answer any two questions)

- 1.a Show that $u = y^3 - 3x^2y$ is harmonic and hence find its harmonic conjugate. (8)
- b Find the image of $\left|z - \frac{1}{2}\right| \leq \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. Also find the fixed points of the transformation $w = \frac{1}{z}$ (7)
- 2.a Define an analytic function and prove that an analytic function of constant modulus is constant. (8)
- b Find the linear fractional transformation that maps $z_1 = 0, z_2 = 1, z_3 = \infty$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively. (7)
- 3.a Show that $f(z) = e^{-x} \cos y - ie^{-x} \sin y$ is differentiable everywhere. Find its derivative. (8)
- b Find the image of the lines $x = c$ and $y = k$, where c & k are constants, under the transformation $w = \sin z$. (7)

PART B

(Answer any two questions)

- 4.a Evaluate $\int_C \operatorname{Re}(z) dz$ where C is a straight line from 0 to $1 + 2i$. (7)
- b Show that $\int_0^\infty \frac{dx}{1+x^4} = \frac{\pi}{2\sqrt{2}}$ (8)
- 5.a Integrate $\frac{z^2}{z^2-1}$ counterclockwise around the circle $|z - 1 - i| = \frac{\pi}{2}$ by Cauchy's Integral Formula. (7)
- b Evaluate $\int_C \frac{z-23}{z^2-4z-5} dz$ where C is $|z - 2 - i| = 3.5$ by Cauchy's Residue Theorem (8)
- 6.a If $f(z) = \frac{1}{z^2}$ find the Taylor series that converges in $|z - i| < R$ and the Laurent's series that converges in $|z - i| > R$. (8)
- b Define three types of isolated singularities with an example for each. (7)

PART C

(Answer any two questions)

7.a Solve by Gauss Elimination:

$$\begin{aligned}x_1 - x_2 + x_3 &= 0, \\-x_1 + x_2 - x_3 &= 0, \\10 x_2 + 25 x_3 &= 90, \\20 x_1 + 10 x_2 &= 80.\end{aligned}\tag{5}$$

b Find the rank. Also find a basis for the row space and column space for

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix}\tag{5}$$

c Find out what type of conic section the quadratic form

$$Q = 17 x^2 - 30 xy + 17 y^2 = 128 \text{ represents and transform it to the principal axes.}\tag{10}$$

8.a Find whether the vectors $[1 \ 2 \ -1 \ 3]$, $[2 \ -13 \ 2]$ and $[-1 \ 8 \ -9 \ 5]$ are linearly dependent. (5)b Show that the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$ is symmetric. Find the spectrum. (5)c Diagonalise $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (10)9. a. Determine whether the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$ is orthogonal? (5)b. Find the Eigen values and Eigen vectors of $\begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$ (5)

c. Define a Vector Space with an example. (10)