

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**FIFTH SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017**

**Course Code: CS309**

**Course Name: GRAPH THEORY AND COMBINATORICS (CS)**

Max. Marks: 100

Duration: 3 Hours

**PART A**

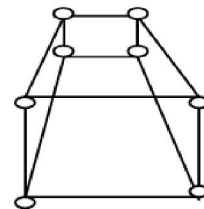
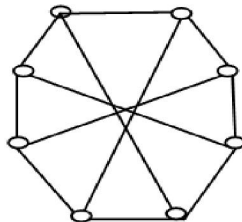
*Answer all questions, each carries 3 marks.*

- |   |   | Marks |
|---|---|-------|
| 1 | Consider a graph $G$ with 4 vertices: $v_1, v_2, v_3$ and $v_4$ and the degrees of vertices are 3, 5, 2 and 1 respectively. Is it possible to construct such a graph $G$ ? If not, why? | (3)   |
| 2 | Draw a disconnected simple graph $G_1$ with 10 vertices and 4 components and also calculate the maximum number of edges possible in $G_1$ .   | (3)   |
| 3 | State Dirac's theorem for hamiltonicity and why it is not a necessary condition for a simple graph to have a Hamiltonian circuit.   | (3)   |
| 4 | Differentiate between symmetric and asymmetric digraphs with examples and draw a complete symmetric digraph of four vertices.   | (3)   |

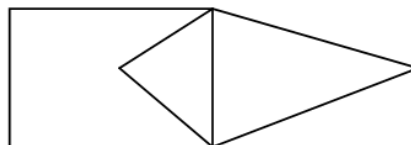
**PART B**

*Answer any two full questions, each carries 9 marks.*

- 5 a) What are the basic conditions to be satisfied for two graphs to be isomorphic? Are the two graphs below isomorphic? Explain with valid reasons (6)



- b) Write any two applications of graphs with sufficient explanation (3)
- 6 a) Consider the graph  $G$  given below: (4)



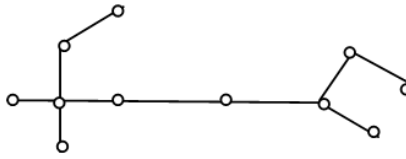
- Define Euler graph. Is  $G$  an Euler? If yes, write an Euler line from  $G$ .
- b) What is the necessary and sufficient condition for a graph to be Euler? And also prove it. (5)
- 7 a) Define Hamiltonian circuits and paths with examples. Find out the number of edge-disjoint Hamiltonian circuits possible in a complete graph with five vertices (5)
- b) State Travelling-Salesman Problem and how  $TSP$  solution is related with Hamiltonian Circuits? (4)

**PART C**

*Answer all questions, each carries 3 marks.*

- 8 List down any two properties of trees and also prove the theorem: *A graph is a tree if and only if it is a minimally connected.* (3)

- 9 Consider the tree T, given below (3)



- 10 Label the vertices of T appropriately and find the center and diameter of T. Prove the statement: (3)

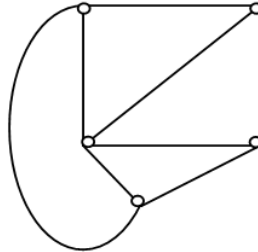
*Every cut-set in a connected graph G must also contain at least one branch of every spanning tree of G*

- 11 List down the properties stating the relationship between the edges of graph G and its dual  $G^*$  (3)

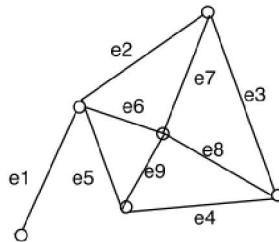
**PART D**

*Answer any two full questions, each carries 9 marks.*

- 12 a) Define spanning trees. Consider the graph G given below and obtain any *three* spanning trees from G. Calculate the number of distinct spanning trees possible from a complete graph with  $n$  vertices. (5)

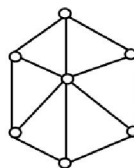


- b) Let  $G = (V, E)$  be a connected graph, and let  $T = (V, S)$  be a spanning tree of G. Let  $e = (a, b)$  be an edge of G *not in* T. Prove that, for any edge  $f$  on the path from  $a$  to  $b$  in T,  $(V, (S \cup \{e\}) - \{f\})$  is another spanning tree for G (4)
- 13 a) Define cut set. Find any four cut sets from the graph G given below and also find the edge connectivity of G. (5)



- b) Define vertex connectivity and draw a graph with an articulation point. (3)
- c) State Euler's Theorem (*formula*). (1)

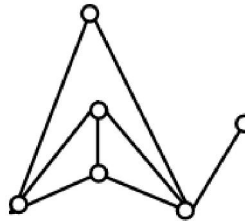
- 14 a) Draw two Kuratowski's graphs and also prove that Kuratowski's first graph is non planar using appropriate inequality. (4)
- b) Draw the geometric dual ( $G^*$ ) of the graph G given below and also check whether G and  $G^*$  are self dual or not, substantiate your answer clearly? (5)



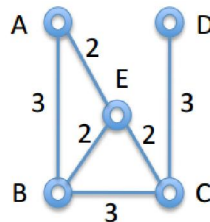
**PART E**

*Answer any four full questions, each carries 10 marks.*

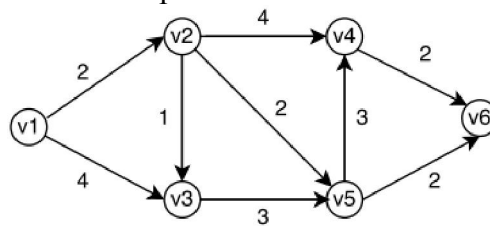
- 15 a) List down any four properties of adjacency matrix (4)  
 b) Construct an adjacency matrix(X) for the following graph and also mention how the concept of edge sequences is described with  $X^3$  (no need to find  $X^3$  from X) (6)



- 16 a) Prove the theorem: (4)  
 If  $A(G)$  is an incidence matrix of a connected graph  $G$  with  $n$  vertices, the rank of  $A(G)$  is  $n-1$   
 b) Describe with examples the usage of incidence matrix to find two graphs ( $g_1$  and  $g_2$ ) are isomorphic. (6)
- 17 a) Define cut-set matrix with an example and list down any four properties of cut-set matrix (6)  
 b) If  $B$  is a circuit matrix of a connected graph  $G$  with  $e$  edges and  $n$  vertices, then show that the number of linearly independent rows in  $B = e-n+1$  (4)
- 18 a) Draw the flow chart of minimum spanning-tree algorithm. (7)  
 b) Find MST from the graph given below by simply applying Kruskal's procedure. (3)



- 19 Write the Dijkstra's shortest path algorithm (no need to draw flowchart). Apply this algorithm to find the shortest path between  $v_1$  and  $v_6$  (10)



- 20 Draw the flowchart of *Connectedness and Components* algorithm and also apply this algorithm on any graph (G) with 2 components. (10)

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