

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**FOURTH SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018**

**Course Code: MA204**

**Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS**  
**(AE, EC)**

Max. Marks: 100

Duration: 3 Hours

*(Normal distribution table is allowed in the examination hall)*

**PART A**

*Answer any two full questions, each carries 15 marks*

Marks

- 1 a) A random variable X has the following probability distribution: (7)

x	-2	-1	0	1	2	3
f(x)	0.1	k	0.2	2k	0.3	3k

Find: i) The value of k                      ii) Evaluate  $P(X < 2)$  and  $P(-2 < X < 2)$

iii) Evaluate the mean of X

- b) The probability that a component is acceptable is 0.93. Ten components are picked at random. What is the probability that: (8)

i) At least nine are acceptable    ii) At most three are acceptable.

- 2 a) Suppose that the length of a phone call in minutes is an exponential random variable with parameter  $\lambda = \frac{1}{10}$ . If someone arrives immediately ahead of you at a public telephone booth, find the probability that you will have to wait: (7)

i) More than 10 minutes            ii) Between 10 and 20 minutes.

- b) For a normally distributed population, 7% of items have their values less than 35 and 89% have their values less than 63. Find the mean and standard deviation of the distribution. (8)

- 3 a) Fit a binomial distribution to the following data and calculate the theoretical frequencies. (8)

x	0	1	2	3	4	5	6	7	8
f	2	7	13	15	25	16	11	8	3

- b) The time between breakdowns of a particular machine follows an exponential distribution, with a mean of 17 days. Calculate the probability that a machine breaks down in a 15 day period. (7)

**PART B**

*Answer any two full questions, each carries 15 marks*

- 4 a) The joint PDF of two continuous random variables X and Y is given by (7)

$$f(x, y) = \begin{cases} kxy & 0 < x < 4, 1 < y < 5 \\ 0 & \text{otherwise} \end{cases}$$

Find: i) k                      ii) The marginal distributions of X and Y

iii) Check whether X and y are independent.

- b) A distribution with unknown mean  $\mu$  has variance equal to 1.5. Use Central Limit Theorem to find how large a sample should be taken from the distribution in order that the probability will be at least 0.95 that the sample mean will be within 0.5 of the population mean. (8)

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- 5 a) The autocorrelation function for a stationary process  $X(t)$  is given by  $R_{XX}(\tau) = 9 + 2e^{-|\tau|}$ . Find the mean value of the random variable  $Y = \int_{\tau=0}^2 X(t)dt$  and the variance of  $X(t)$ . (7)
- b) A random process  $X(t)$  is defined by  $X(t) = Y(t) \cos(\omega t + \theta)$  Where  $Y(t)$  is a WSS process,  $\omega$  is a constant and  $\theta$  is a random variable which is uniformly distributed in  $[0, 2\pi]$  and is independent of  $Y(t)$ . Show that  $X(t)$  is WSS. (8)
- 6 a) Consider the random process  $X(t) = A \cos(\omega t + \theta)$  where  $A$  and  $\omega$  are constants and  $\theta$  is a uniformly distributed random variable in  $(0, 2\pi)$ . Check whether or not the process is WSS. (7)
- b) The joint PDF of two continuous random variables  $X$  and  $Y$  is (8)
- $$f(x, y) = \begin{cases} 8xy, & 0 < y < x < 1 \\ 0, & \text{otherwise} \end{cases}$$
- i) Check whether  $X$  and  $Y$  are independent      ii) Find  $P(X + Y < 1)$

**PART C**

*Answer any two full questions, each carries 20 marks*

- 7 a) The number of particles emitted by a radioactive source is Poisson distributed. The source emits particles at a rate of 6 per minute. Each emitted particle has a probability of 0.7 of being counted. Find the probability that 11 particles are counted in 4 minutes. (4)
- b) Assume that a computer system is in any one of the three states: busy, idle and under repair, respectively, denoted by 0,1,2. Observing its state at 2 P. M. each day, the transition probability matrix is  $P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix}$  (8)
- Find out the third step transition probability matrix and determine the limiting probabilities.
- c) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between two consecutive arrivals is: (8)
- i) More than 1 minute      ii) Between 1 minute and 2 minutes  
iii) Less than or equal to 4 minutes.
- 8 a) Use Trapezoidal rule to evaluate  $\int_0^1 x^3 dx$  considering five subintervals (4)
- b) Using Newton's forward interpolation formula, find  $y$  at  $x = 8$  from the following table: (8)
- |     |   |    |    |    |    |    |
|-----|---|----|----|----|----|----|
| x : | 0 | 5  | 10 | 15 | 20 | 25 |
| y : | 7 | 11 | 14 | 18 | 24 | 32 |
- c) Using Euler's method, solve for  $y$  at  $x = 0.1$  from  $\frac{dy}{dx} = x + y + xy$ ,  $y(0) = 1$  taking step size  $h = 0.025$ . (8)
- 9 a) The transition probability matrix of a Markov chain  $\{X_n, n \geq 0\}$  having three states 1, 2 and 3 is  $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}$  and the initial probability distribution is  $p(0) = [0.5 \ 0.3 \ 0.2]$ . Find the following: (10)
- i)  $P\{X_2 = 2\}$       ii)  $P\{X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 3\}$ .
- b) Using Newton-Raphson method, compute the real root of  $f(x) = x^3 - 2x - 5$  correct to 5 decimal places. (5)
- c) Using Lagrange's interpolation formula, find the values of  $y$  when  $x = 10$  from the following table : (5)
- |     |    |    |    |    |
|-----|----|----|----|----|
| x : | 5  | 6  | 9  | 11 |
| y : | 12 | 13 | 14 | 16 |

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