

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017

Course Code: CS201

Course Name: DISCRETE COMPUTATIONAL STRUCTURES (CS, IT)

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

Marks

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| 1 | Assume $A = \{1, 2, 3\}$ and $\rho(A)$ be its power set. Let \subseteq be the subset relation on the power set. Draw the Hasse diagram of $(\rho(A), \subseteq)$ | (3) |
| 2 | Let R denote a relation on the set of ordered pairs of positive integers such that $(x, y)R(u, v)$ iff $xv = yu$. Show that R is an equivalence relation | (3) |
| 3 | Prove that in any group of six people, at least three must be mutual friends or at least three must be mutual strangers. | (3) |
| 4 | Define GLB and LUB for a partially ordered set. Give an example | (3) |

PART B

Answer any two full questions, each carries 9 marks.

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| 5 | a) Suppose $f(x) = x + 2, g(x) = x - 2$ and $h(x) = 3x$ for $x \in \mathbb{R}$, where \mathbb{R} is the set of real numbers. Find $g \circ f, f \circ g, f \circ f, g \circ g, f \circ h, h \circ g, h \circ h$ and $(f \circ h) \circ g$ | (4) |
| | b) Prove that every equivalence relation on a set generates a unique partition of the set with the blocks as R -equivalence classes | (5) |
| 6 | a) Show that the set \mathbb{N} of natural numbers is a semigroup under the operation $x * y = \max(x, y)$. Is it a monoid? | (3) |
| | b) Solve the recurrence relation $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2 - 2r + 1$ | (6) |
| 7 | a) Show that for any commutative monoid $\langle M, * \rangle$, the set of idempotent elements of M forms a submonoid. | (5) |
| | b) Define subsemigroups and submonoids. | (4) |

PART C

Answer all questions, each carries 3 marks.

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| 8 | Show that, for an abelian group, $(a * b)^{-1} = a^{-1} * b^{-1}$ | (3) |
| 9 | Show that every chain is a distributive lattice. | (3) |
| 10 | Simplify the Boolean expression $a'b'c + ab'c + a'b'c'$ | (3) |
| 11 | Let $G = \{1, a, a^2, a^3\}$ ($a^4 = 1$) be a group and $H = \{1, a^2\}$ is a subgroup of G under multiplication. Find all cosets of H . | (3) |

PART D

Answer any two full questions, each carries 9 marks.

- 12 a) Show that the order of a subgroup of a finite group divides the order of the group. (6)
 b) Define ring homomorphism. (3)
- 13 Show that (I, \oplus, \otimes) is a commutative ring with identity, where the operations \oplus and \otimes are defined, for any $a, b \in I$, as $a \oplus b = a + b - 1$ and $a \otimes b = a + b - ab$. (9)
- 14 a) Let (L, \leq) be a lattice and $a, b, c, d \in L$. Prove that if $a \leq c$ and $b \leq d$, then (5)
 (i) $a \vee b \leq c \vee d$
 (ii) $a \wedge b \leq c \wedge d$
 b) Show that in a Boolean algebra, for any a, b, c (4)
 $(a \wedge b \wedge c) \vee (b \wedge c) = b \wedge c$

PART E

Answer any four full questions, each carries 10 marks.

- 15 a) a) Construct truth table for $(\sim p \wedge (\sim q \wedge r)) \vee ((q \wedge r) \vee (p \wedge r))$ (6)
 b) Explain proof by Contrapositive with example. (4)
- 16 Prove the following implication (10)
 $(x)(P(x) \vee Q(x)) \implies (x) P(x) \wedge (\exists x) Q(x)$
- 17 a) Represent the following sentences in predicate logic using quantifiers (6)
 (i) "x is the father of the mother of y"
 (ii) "Everybody loves a lover"
 b) Determine whether the conclusion C follows logically from the premises (4)
 $H_1: \sim p \vee q, H_2: \sim(q \wedge \sim r), H_3: \sim r$ C: $\sim p$
- 18 a) Without using truth table prove $p \rightarrow (q \rightarrow p) \iff \sim p \rightarrow (p \rightarrow q)$ (4)
 b) Determine the validity of the following statements using rule CP. (6)
 "my father praises me only if I can be proud of myself. Either I do well in sports or I can't be proud of myself. If I study hard, then I can't do well in sports. Therefore if my father praises me then I do not study well"
- 19 a) Show that $r \rightarrow s$ can be derived from the premises $p \rightarrow (q \rightarrow s), \sim r \vee p, q$ (4)
 b) Prove, by Mathematical Induction, that $n(n+1)(n+2)(n+3)$ is divisible by 24, for all natural numbers n (6)
- 20 a) "If there are meeting, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. There was no meeting". Show that these statements constitute a valid argument. (6)
 b) Show that $2^n < n!$ For $n \geq 4$ (4)
