

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
THIRD SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

**Course Code: MA201**

**Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS**

Max. Marks: 100

Duration: 3 Hours

**PART A**

*Answer any two full questions, each carries 15 marks*

Marks

- 1 a) Prove that  $f(z) = e^{x+iy}$  is analytic. Find  $f'(z)$ . (7)
- b) Show that  $v = 3x^2y - y^3$  is harmonic. Also find the harmonic conjugate of  $v$ . (8)
- 2 a) Find the linear fractional transformation that maps  $z_1 = 0, z_2 = 1, z_3 = \infty$  (8)  
onto  $w_1 = -1, w_2 = -i, w_3 = 1$  respectively.
- b) Find the image of the lines  $x = a$  and  $y = b$  where  $a$  and  $b$  are constants, under (7)  
the transformation  $w = z^2$
- 3 a) If  $f(z) = u + iv$  is analytic, prove that  $u = c_1$  and  $v = c_2$  are families of curves (7)  
cutting orthogonally.
- b) Prove that  $w = \frac{z-i}{1-iz}$  maps the upper half plane ( $y > 0$ ) into the interior of  $|w| = 1$  (8)

**PART B**

*Answer any two full questions, each carries 15 marks*

- 4 a) Expand  $f(z) = \frac{1}{z^2}$  as Taylor's series about  $z = 2$  (7)
- b) Evaluate  $\oint_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)} dz$  where  $C$  is  $|z| = 3$ , using Cauchy's integral formula. (8)
- 5 a) Evaluate  $\oint_C \frac{z-23}{z^2-4z-5} dz$  where  $C: |z-2-i| = 3.2$ , using Cauchy's residue (7)  
theorem.
- b) Show that  $\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^2} dx = \frac{3\pi}{8}$  (8)
- 6 a) Find the Laurent's series expansion of  $f(z) = \frac{e^{2z}}{(z+1)^2}$  about  $z = -1$  (7)
- b) Find the poles and residues of the function  $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$  (8)

## PART C

*Answer any two full questions, each carries 20 marks*

7 a) Find the Eigen value and Eigen vector of the matrix  $A = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$  (8)

b) Find the rank of the matrix  $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}$  (6)

c) Solve the system of equations  $x - y + z = 0, -x + y - z = 0,$   
 $10y + 25z = 90, 20x + 10y = 80$  (6)

8 a) Find out what type of conic section the quadratic form  
 $q = 3x_1^2 + 21x_1x_2 + 3x_2^2 = 0$  represents. (8)

b) Show that the matrix  $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & \sqrt{3} \\ 0 & \sqrt{3} & 1 \end{bmatrix}$  is orthogonal. (6)

c) Show that the system of equations are inconsistent. (6)  
 $2x + 6y = -11, \quad 6x + 20y - 6z = -3, \quad 6y - 18z = -1$

9 a) (i) Show that the vectors  $(1, -1, 0), (1, 3, -1)$  and  $(5, 3, -2)$  are linearly dependent. (8)

(ii) Show that the quadratic form  $4x_1^2 + 12x_1x_2 + 13x_2^2 = 16$  is positive definite.

b) Diagonalize the matrix  $A = \begin{bmatrix} 6 & 0 & 0 \\ 12 & 2 & 0 \\ 21 & -6 & 9 \end{bmatrix}$  (12)

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