Reg No.:_____   Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third semester B.Tech degree examinations (S) September 2020

Course Code: MA201

Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100   Duration: 3 Hours

PART A

Answer any two full questions, each carries 15 marks

Marks

1 a) Find out and give reason whether $f(z)$ is continuous at $z = 0$ (7)

$$f(z) = \begin{cases} \dfrac{Re\ z}{1 - |z|} & ,z \neq 0 \\ 0, & z = 0 \end{cases}$$

   b) Determine a so that $u = e^{-\pi x} cos ay$ is harmonic and then find the harmonic conjugate.   (8)

2 a) Determine the region of the w-plane into which the triangle formed by $x = 1, y = 1\ and\ x + y = 1$ is mapped under the transformation $w = z^2$ (7)

   b) Check whether $f(z) = e^z$ is analytic everywhere .   (8)

3 a) Find the image of $-\frac{1}{2} \leq x \leq \frac{1}{2}, -\pi < y < \pi$ under $w = e^z$ (7)

   b) Find the linear fractional transformation that maps $0,1,2$ onto $1,\frac{1}{2},\frac{1}{3}$ (8)

PART B

Answer any two full questions, each carries 15 marks

4 a) Evaluate $\int_0^{4+2i} \bar{z} dz$ along the curve given by $z = t^2 + it$ (7)

   b) Evaluate $\int_C \frac{2z-1}{z^2-z} dz$ along the curve C: $|z| = 3$ using Cauchy's integral formula.   (8)

5 a) Find the Laurent's series expansion of $f(z) = \frac{1}{z^2+3z+2}$ in the region $1 < |z| < 2$ (7)

   b) Find all singularities and the corresponding residues (i) $\frac{8}{1+z^2}$ (ii) $tan z$ (8)

6 a) Evaluate $\int_C \frac{e^z}{\cos \pi z} dz$ where c is the unit circle $|z|=1$ using Residue Theorem.   (7)

   b) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+cos\theta}$ (8)

PART C

Answer any two full questions, each carries 20 marks

7 a) Solve by Gauss elimination   (8)

$$5x - 6y + 4z = 15, 7x + 4y - 3z = 19, 2x + y + 6z = 46$$

b) Find the rank of $\begin{bmatrix} 6 & 0 & -2 & 0 \\ 0 & -1 & -1 & 5 \\ 2 & -1 & -1 & 0 \end{bmatrix}$ (6)

c) Let $V = \{(v_1, v_2, v_3) \in R^3 : 3v_2 + v_3 = 2\}$. Is V a vector space under the usual operations in R^3? (6)

8 a) Find the eigen values and eigen vectors of $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (10)

b) Is the matrix A $= \dfrac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$ orthogonal? (5)

c) Check whether $\{(2,0,0,7),(2,0,0,8),(2,0,0,9),(2,0,1,0)\}$ are linearly independent in R^4 (5)

9 a) Diagonalize $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ (8)

b) Transform to principal axis and find what kind of conic section is given by the quadratic form $4x^2 + 12xy + 13y^2 = 16$ (12)

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