

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth semester B.Tech examinations (S), September 2020

Course Code: MA202**Course Name: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS**

Max. Marks: 100

Duration: 3 Hours

*Normal distribution table is allowed in the examination hall.***PART A (MODULES I AND II)***Answer two full questions.*

- 1 a) Let X be a discrete random variable with mean 10 and variance 25. Find the positive values of α and β such that $Y = \alpha X - \beta$ has mean 0 and variance 1. 7
- b) Derive the mean and variance of a Poisson Distribution. 8
- 2 a) If a continuous random variable has the probability distribution function 7
- $$f(x) = \begin{cases} ke^{-3x} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$
- then find (i) value of k (ii) $P[0 \leq X \leq 2]$ (iii) $P[X > 1.5]$
- b) In a Normal Distribution, if 6% of the items are below 60 and 39% are above 70, then find the mean and standard deviation. 8
- 3 a) Out of 2000 families with 4 children each, how many would you expect to have (i) at least one boy (ii) at most one boy 7
- b) If X follows a uniform distribution in $(-2, 2)$, then (i) find $P[|X - 1| \leq 2]$ (ii) find k for which $P[X > k] = \frac{1}{3}$ (iii) Distribution function 8

PART B (MODULES III AND IV)*Answer two full questions.*

- 4 a) Find the Fourier Sine Integral of $f(x) = \begin{cases} \sin x & \text{if } 0 \leq x \leq \pi \\ 0 & \text{if } x > \pi \end{cases}$ 7
- b) Find the Fourier Cosine Transform of $f(x) = e^{-4x}$. Hence deduce that 8
- $$\int_0^{\infty} \frac{\cos 2x}{x^2 + 16} dx = \frac{\pi}{8} e^{-8}$$
- 5 a) Using Convolution theorem, evaluate the Inverse Laplace Transform of $\frac{s}{(s^2 + 4)^2}$ 7
- b) Evaluate (i) $L[t \sin^2 2t]$ (ii) $L^{-1} \left[\frac{s+5}{s^2 + 4s + 13} \right]$ 8

- 6 a) Find the Fourier Transform of $f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ Hence show that $\int_0^{\infty} \frac{\sin \omega}{\omega} d\omega = \frac{\pi}{2}$ 7

- b) Solve using Laplace Transform: $y'' - 3y' + 2y = 4$ given $y(0) = 2, y'(0) = 3$ 8

PART C (MODULES V AND VI)

Answer two full questions.

- 7 a) Using Lagrange's interpolation formula, find a parabola of the form $y = ax^2 + bx + c$ passing through the points (0,0), (2,4) and (3,12) 6

- b) Using Newton-Raphson Method, find the real root lying between 0 and 1 of $3x - \cos x - 1 = 0$. (Correct to three decimal places) 7

- c) Apply Lagrange's interpolation formula to find y at $x = 2$ for the following values for $y = f(x)$. Given $f(0) = -12, f(1) = 0, f(3) = 6$ and $f(4) = 12$. 7

- 8 a) Solve by Gauss Elimination Method: 6

$$3x + 4y + 5z = 18, \quad 2x - y + 8z = 13, \quad 5x - 2y + 7z = 20.$$

- b) Evaluate $I = \int_0^6 \frac{1}{1+x} dx$ using (i) Trapezoidal Rule (ii) Simpson's $\frac{1}{3}$ Rule (Take $h=1$). Also find the value of the integral by actual integration. 7

- c) Using Euler's Method compute the value of $y(0.1)$ given $y' = x + \frac{1}{y}, y(0) = 1$ (Take $h = 0.025$) 7

- 9 a) Using Newton's Interpolation Formula find $f(1.2)$ and $f(2.0)$ from the table. 10

x	1	1.4	1.8	2.2
$y = f(x)$	3.49	4.82	5.96	6.50

- b) Using Runge - Kutta Method of 4th order, find $y(0.8)$ correct to four decimal places if $\frac{dy}{dx} = y - x^2$ given $y(0.6) = 1.7379$ (Take $h = 0.1$) 10