

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third Semester B.Tech Degree (S,FE) Examination December 2020 (2015 scheme)

Course Code: MA201**Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS**

Max. Marks: 100

Duration: 3 Hours

PART A*Answer any two full questions, each carries 15 marks*

Marks

- 1 a) Show that the function $f(z)$ defined by. (7)

$$f(z) = \begin{cases} \frac{\operatorname{Re}(z)}{z} & z \neq 0 \\ 0, & z = 0 \end{cases} \text{ is not continuous at } z = 0.$$

- b) Show that $u = x^3 - 3xy^2$ is harmonic. Hence find its harmonic conjugate. (8)
- 2 a) Determine the Linear fractional transformation that maps $z_1 = 0, z_2 = 2i, z_3 = -2i$ onto $w_1 = -1, w_2 = 0, w_3 = \infty$ respectively. (8)
- b) Find the image of the strip $\frac{1}{2} \leq x \leq 1$ under the transformation $w = z^2$. (7)
- 3 a) Show that $f(z) = z^2$ is analytic everywhere and find its derivative. (7)
- b) Under the transformation $w = \frac{1}{z}$, find the image of $x \geq 1$. (8)

PART B*Answer any two full questions, each carries 15 marks*

- 4 a) Evaluate $\int_C \operatorname{Re}(z) dz$ where C is the parabola $y = 1 + \frac{1}{2}(x-1)^2$ from $1+i$ to $3+3i$ (7)
- b) Use Cauchy's integral formula to evaluate $\int_C \frac{e^z \cos z}{(z-\frac{\pi}{4})^3} dz$ where C is the unit circle counterclockwise. (8)
- 5 a) Find the poles and residues of $f(z) = \frac{e^z}{z^2 + \pi^2}$. (7)
- b) Find the Taylor series and Laurent series expansions of $f(z) = \frac{1}{1+z}$ about $z = -i$ (8)
- 6 a) Evaluate using Cauchy's residue theorem $\int_C \tan 2\pi z dz$ where C is the circle $|z - 0.2| = 0.2$ (7)

b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2+2}{(x^2+4)(x^2+9)} dx$ (8)

PART C

Answer any two full questions, each carries 20 marks

7 a) Examine whether the vectors $(1, 2, 3, 4)$, $(2, 0, 1, -2)$ & $(3, 2, 4, 2)$ are linearly independent or not. (6)

b) Solve the system of equations $y + z = -2$, $4y + 6z = -12$, $x + y + z = 2$ by Gauss elimination method. (7)

c) Find a basis for row space and a basis for column space of $\begin{bmatrix} 6 & 0 & -3 & 0 \\ 0 & -1 & 0 & 5 \\ 2 & 0 & -1 & 0 \end{bmatrix}$ (7)

8 a) Determine whether the matrix $A = \begin{bmatrix} 1/3 & 2/3 & -2/3 \\ -2/3 & 2/3 & 1/3 \\ 2/3 & 1/3 & 2/3 \end{bmatrix}$ is orthogonal. Is A symmetric? (6)

b) Examine the definiteness of the quadratic form $q = 2xy + 2yz + 2xz$. (7)

c) Find the Eigen values and Eigen vectors of $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ (7)

9 a) Determine the values of λ for which the following system of linear equations possesses a non-trivial solution (5)

$$3x + y - \lambda z = 0 ; 4x - 2y - 3z = 0 ; 2\lambda x + 4y + \lambda z = 0$$

b) Diagonalize the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$ (10)

c) Find the rank of the following matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ (5)
