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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third Semester B.Tech (minor) Degree Examination December 2020

Course Code: MAT281

Course Name: ADVANCED LINEAR ALGEBRA

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1 Define subspace. List all subspaces of R^3.

2 The set $B = \{1+t, 1+t^2, t+t^2\}$ is a basis for P_2. Find the coordinate vector of $p(t) = 6 + 3t - t^2$ relative to B.

3 Define isomorphism between vector spaces. Give one example.

4 $T: R^2 \to R^3$ defined by $T(x_1, x_2) = (3x_1 + x_2, 5x_1 + 7x_2, x_1 + 3x_2)$ is a linear transformation. Find the Standard matrix of the transformation.

5 Find a nonzero vector w that is orthogonal to $u = (1,2,1)$ and $v = (2,5,4)$ in R^3

6 If u and v are vectors in an inner product space V show that $\|u+v\| \le \|u\| + \|v\|$

7 Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$ and $u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$. Show that u is an eigenvector of A.

8 Find the characteristic polynomial for the linear transformation $T: R^2 \to R^2$ defined by $T(x, y) = (3x + 5y, 2x - 7y)$

9 Find the equation $y = \beta_0 + \beta_1 x$ of the least squares line that best fits the data points $(2,1)$, $(5,2)$, $(7,3)$, $(8,3)$

10 If $u_1 = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$, $u_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$, $y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $W = span\{u_1, u_2\}$, find the closest point in W to y

$(10 \times 3 = 30)$

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11 (a) Find bases for the row space, the column space and null space of the matrix

$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$$ (7)

(b) Let H and K are subspaces of a vector space V. Show that $H \cap K$ is also a Subspace of V. Give an example to show that the union of subspaces, is not in general a subspace. (7)

12 (a) For what value of h will y be in the subspace of R^3 spanned by v_1, v_2, v_3 if

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}, v_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} \text{ and } y = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix}$$ (7)

(b) Use coordinate vectors to test whether the set of polynomials span P_2.Justify your conclusion.

$$1 - 3t + 5t^2, -3 + 5t - 7t^2, -4 + 5t - 6t^2, 1 - t^2$$ (7)

Module 2

13 (a) Show that $T: R^3 \to R^3$ defined by $T(x, y, z) = (x, y, 0)$ is a linear transformation. Find kernel of T and range of T. (7)

(b) The mapping $T: P_2 \to P_2$ defined by $T(a_0 + a_1 t + a_2 t^2) = a_1 + 2a_2 t$ is a linear transformation. Find the B- matrix for T, when $B = \{1, t, t^2\}$. (7)

14 (a) Let $b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, b_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}, c_1 = \begin{bmatrix} -7 \\ 9 \end{bmatrix}, c_2 = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$ and consider the bases for R^2 given by $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$. Find the change of coordinate matrix from C to B and the change of coordinate matrix from B to C. (7)

(b) Define $T: R^2 \to R^2$ by $T(x, y) = (x - y, x - 2y)$.Determine whether T is one to one and onto if so find $T^{-1}(x, y)$. (7)

Module 3

15 (a) Find an orthonormal basis for the column space of the matrix

$$A = \begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}$$ (7)

(b) If $C[a, b]$ is the vector space of all continuous functions on $[a, b]$,show that $< f, g > = \int_a^b f(t)g(t)dt$ defines an inner product on $[a, b]$. (7)

16 (a) Let $u_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ -2 \end{bmatrix}, u_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, y = \begin{bmatrix} 4 \\ 3 \\ 3 \\ -1 \end{bmatrix}$ and

$W = span\{u_1, u_2, u_3\}$. Show that $\{u_1, u_2, u_3\}$ is an orthogonal basis for W and express y as the sum of a vector in W and a vector orthogonal to W (7)

(b) Let W be the subspace of R^5 spanned by the vectors $w_1 = (2,2,-1,0,1)$, $w_2 = (-1,-1,2,-3,1), w_3 = (1,1,-2,0,-1), w_4 = (0,0,1,1,1)$.
Find a basis for the orthogonal complement of W. (7)

Module 4

17 (a) Diagonalize the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$ (7)

(b) Define $T: R^2 \to R^2$ by $T(x) = Ax$ where $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a basis B for R^2 with the property that $B - matrix$ for T is a diagonal matrix. (7)

18 (a) Apply the power method to $A = \begin{bmatrix} 6 & 5 \\ 1 & 2 \end{bmatrix}$ with $x_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and estimate the dominant eigenvalue and a corresponding eigenvector of A. (7)

(b) Find an orthogonal matrix P that diagonalizes $A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$. (7)

Module 5

19 (a) Solve the equation $Ax = b$ using the LU factorization method, where

$$A = \begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix}, \; b = \begin{bmatrix} -7 \\ 5 \\ 2 \end{bmatrix}$$ (7)

(b) Find a least-squares solution of the inconsistent system $Ax = b$ for

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, \; b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$$ (7)

20 (a) Find a QR factorization of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (7)

(b) Find a singular value decomposition of the matrix $A = \begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$ (7)

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