Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third Semester B.Tech Degree Examination December 2020 (2019 Scheme)

Course Code: MAT203

Course Name: Discrete Mathematical Structures

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1 Without using truth tables, show that (3)

$p \rightarrow (q \rightarrow r) \equiv p \rightarrow (\sim q \vee r) \equiv (p \wedge q) \rightarrow r$

2 Define the terms: Converse, Inverse and Contrapositive. (3)

3 What is Pigeonhole Principle? Given a group of 100 people, at minimum, how many people were born in the same month? (3)

4 In how many ways can the letters of the word 'MATHEMATICS' be arranged such that vowels must always come together? (3)

5 If $A = \{1,2,3,4\}$, give an example of a relation on A which is reflexive and transitive, but not symmetric. (3)

6 Define a complete lattice. Give an example. (3)

7 Define a recurrence relation. Give an example. (3)

8 Determine the coefficient of x^{15} in $f(x) = (x^2 + x^3 + x^4 + \ ...)^4$ (3)

9 Define semi-group. Give an example. (3)

10 Show that the set of idempotent elements of any commutative monoid forms a submonoid. (3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11(a) Check whether the propositions $p \wedge (\sim q \vee r)$ and $p \vee (q \wedge \sim r)$ are logically equivalent or not. (6)

(b) Check the validity of the statement (8)

$p \rightarrow q$

$q \rightarrow (r \wedge s)$

$\sim r \vee (\sim t \vee u)$

$p \wedge t$

$\therefore u$

12(a) Show that $\big((p \to q) \wedge (q \to r)\big) \to (p \to r)$ is a tautology. (6)

(b) Let p, q, r be the statements given as (8)

p: Arjun studies. q: He plays cricket. r: He passes Data Structures.

Let p_1, p_2, p_3 denote the premises

p_1: If Arjun studies, then he will pass Data Structures.

p_2: If he doesn't play cricket, then he will study.

p_3: He failed Data Structures.

Determine whether the argument $(p_1 \wedge p_2 \wedge p_3) \to q$ is valid.

Module 2

13(a) State Binomial theorem. Find the coefficient of xyz^2 in $(2x - y - z)^4$ (6)

(b) Determine the number of positive integers n such that $1 \le n \le 100$ and n is not divisible by 2,3 or 5. (8)

14(a) Prove that if 7 distinct numbers are selected from $\{1,2,3, \dots, 11\}$, then sum of two among them is 12. (6)

(b) An urn contains 15 balls, 8 of which are red and 7 are black. In how many ways can 5 balls be chosen so that (i)all the five are red (ii)all the five are black (iii) 2 are red and 3 are black (iv)3 are red and 2 are black. (8)

Module 3

15(a) If f, g and h are functions on integers, $f(n) = n^2, g(n) = n + 1$, $h(n) = n - 1$, then find (i) $f^\circ g^\circ h$ (ii) $g^\circ f^\circ h$ (iii) $h^\circ f^\circ g$ (6)

(b) If $A = \{a, b, c\}$ and $P(A)$ be its power set. The relation $\le$ be the subset relation defined on the power set. Draw the Hasse diagram of $(P(A), \le)$. (8)

16(a) Let R be a relation on Z by xRy if $4|(x - y)$. Then find all equivalence classes. (6)

(b) Find the complement of each element in D_{42}. (8)

Module 4

17(a) Solve the recurrence relation $a_{n+1} = 2a_n + 1$, $n \ge 0$, $a_0 = 0$. (6)

(b) Solve the recurrence relation $a_{n+2} = a_{n+1} + a_n$, $n \ge 0$, $a_0 = 0, a_1 = 1$ (8)

18(a) Solve the recurrence relation $a_{n+2} - 4a_{n+1} + 3a_n = -200$, $n \geq 0$, $a_0 = 3000, a_1 = 3300$ (6)

(b) Solve the recurrence relation $a_n = 2a_{n-1} - 4a_{n-2}$, $n \geq 3, a_1 = 2, a_2 = 0$ (8)

Module 5

19(a) If $f: (R^+, \circ) \rightarrow (R, +)$ as $f(x) = lnx$, where R^+ is the set of positive real numbers. Show that f is a monoid isomorphism from R^+ onto R. (6)

(b) Show that every subgroup of a cyclic group is cyclic. (8)

20(a) State and prove Lagrange's Theorem. (6)

(b) If $A = \{1,2,3\}$. List all permutations on A and prove that it is a group. (8)