01MAT101121903-B

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Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

First Semester B.Tech Degree Regular and Supplementary Examination December 2020

(2019 Scheme)

Course Code: MAT101
Course Name: LINEAR ALGEBRA AND CALCULUS
(2019 Scheme)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks. Marks

1 Find the rank of the matrix $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 2 & 4 & 1 \\ 5 & 6 & 7 & 5 \end{bmatrix}$ (3)

2 What type of conic section the following quadratic form represent?
$Q = 17x_1^2 - 30x_1x_2 + 17x_2^2 = 128$ (3)

3 If $U = \frac{x^3 + y^3}{x - y}$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y}$. (3)

4 If $z = x^2y;\ x = t^2,\ y = t^3$ find $\frac{dz}{dt}$ using chain rule. (3)

5 Evaluate $\int_0^3 \int_0^2 \int_0^1 xyz\, dx\, dy\, dz$ (3)

6 Use double integrals to find the volume of the solid enclosed below the plane $z = 4 - x - y$ and above the rectangle $R = \{(x,y);\ 0 \le x \le 1,\ 0 \le y \le 2\}$. (3)

7 Does the series $\sum_{k=1}^{\infty} \left(\frac{4}{5}\right)^k$ converge? If so, find the sum. (3)

8 Test the convergence of the series $\sum_{k=1}^{\infty} \frac{k^2}{2k^2 - 1}$ (3)

9 Find the binomial series for $f(x) = \frac{1}{\sqrt{1+x}}$ up to third degree term. (3)

10 Find the Maclaurin's series for $f(x) = x\cos x$ up to third degree term. (3)

PART B

Answer one full question from each module, each question carries 14 marks

Module-I

11 a) Using Gauss elimination method find the solution of the system $x+y-z=9, 8y+6z=-6, -2x+4y-6z=40$ (7)

b) Find the matrix of transformation that diagonalize the matrix

$\begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$. Also, find the diagonal matrix. (7)

12 a) Find the value of λ and μ for which the system of equations

$2x+3y+5z = 9 \quad 7x+3y-2z = 8 \quad 2x+3y+\lambda z = \mu$ (7)

has (a) no solution (b) unique solution (c) more than one solution

b) Find the eigen values and eigen vectors for the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (7)

Module-II

13 a) If $w=\sqrt{x^2+y^2+z^2}$, $x=\cos\theta,\ y=\sin\theta,\ z=\tan\theta,$

find $\frac{dw}{d\theta}$ at $\theta=\frac{\pi}{4}$ (7)

b) Find the local linear approximation L of $f(x,y,z)=xyz$ at the point $P(1,2,3)$. Compute the error in approximation f by L at the point $Q(1.001,2.002,3.003)$. (7)

14 a) Locate all relative extrema of $f(x,y)=x^3y^2(12-x-y)$ (7)

b) Let f be a differentiable function of three variables and suppose that $w=f(x-y,y-z,z-x)$, show that $\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z}=0$ (7)

Module-III

15 a) Find the area bounded by the $x-axis,\ y=2x,\ x+y=1$. (7)

b) Change the order of integration and hence evaluate $\int_0^1 \int_{x^2}^{2-x} dy\,dx$ (7)

16 a) Find the volume bounded by the cylinder $x^2+y^2=9$ and the planes $y+z=3$ and $z=0$ (7)

b) Find the mass and centre of gravity of the lamina in the first quadrant bounded by the circle $x^2 + y^2 = 1$ and the coordinate planes with density function xy. (7)

Module-IV

17 a) Test the convergence $i)\sum_{k=1}^{\infty}\frac{k(k-1)}{(k+1)(k+2)(k+3)}$ $ii)\sum_{k=1}^{\infty}\left(\frac{k+2}{2k-1}\right)^k$ (7)

b) Test whether the following series is absolutely convergent or conditionally convergent. $\sum_{k=1}^{\infty}\frac{(-1)^k}{\sqrt{k(k+1)}}$ (7)

18 a) Test the convergence of the series $1+\frac{1.3}{1.2}+\frac{1.3.5}{1.2.3}+\frac{1.3.5.7}{1.2.3.4}+\ldots$ (7)

b) Test the convergence: $(i)\sum_{k=1}^{\infty}\frac{2}{3^k+5}$ $(ii)\sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{k}{2k+3}\right)$ (7)

Module-V

19 a) Find the Fourier series of $f(x)=\begin{cases}1+\frac{2x}{\pi}, -\pi<x<0\\ 1-\frac{2x}{\pi}, 0<x<\pi\end{cases}$ (7)

b) Obtain Fourier series of e^x the interval $(-1,1)$ (7)

20 a) Find the Fourier series $f(x)=x^2-2$ in the interval $(-2,2)$ (7)

b) Find the Fourier cosine series of $f(x) = x^2$ in $0 < x < \pi$. (7)

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