

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

First Semester B.Tech Degree Regular and Supplementary Examination December 2023 (2019 scheme)

Course Code: MAT 101**Course Name: LINEAR ALGEBRA AND CALCULUS****(2019 -Scheme)**

Max. Marks: 100

Duration: 3 Hours

PART A*Answer all questions, each carries 3 marks*

- | | | Marks |
|----|--|-------|
| 1 | Find the rank of the matrix $\begin{bmatrix} 3 & 5 & 0 \\ 0 & 0 & 5 \\ 5 & 0 & 0 \end{bmatrix}$ | (3) |
| 2 | Show that the quadratic form $Q = 3x_1^2 + 22x_1x_2 + 3x_2^2$ is indefinite. | (3) |
| 3 | Find $f_x(1,3)$ and $f_y(1,3)$ if $f(x,y) = 2x^3y^2 + 2y + 4x$. | (3) |
| 4 | If $z = x^2y^2$ where $x = t^4, y = t^3$ find $\frac{dz}{dt}$ using chain rule. | (3) |
| 5 | Evaluate $\int_0^1 \int_0^2 (x+5)dydx$. | (3) |
| 6 | Evaluate $\iint_R \frac{\sin x}{x} dA$, where R is the triangular region bounded by the x-axis, $y = x$ and $x = 1$. | (3) |
| 7 | Determine whether the series $\sum_{k=1}^{\infty} \left(-\frac{3}{5}\right)^k$ converges and if so find its sum | (3) |
| 8 | Examine the convergence of $\sum_{k=1}^{\infty} \frac{4^k}{k^2}$ | (3) |
| 9 | Find the Taylor series expansion of $f(x) = \frac{1}{x}$ about $x = -1$ | (3) |
| 10 | Find the half range sine series representation of $f(x) = k$ in $(0, \pi)$ | (3) |

PART B*Answer one full question from each module, each question carries 14 marks.***MODULE 1**

- 11 a Solve the following linear system of equations using Gauss elimination method (7)
- $$\begin{aligned} x + y - z &= 9 \\ 8y + 6z &= -6 \\ -2x + 4y - 6z &= 40 \end{aligned}$$
- b Find the eigenvalues and eigenvectors of $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ (7)

- 12 a Solve the following linear system of equations using Gauss elimination method. (7)
- $$\begin{aligned} 3x - 11y - 2z &= -6 \\ 4y + 4z &= 24 \\ 6x - 17y + z &= 18 \end{aligned}$$

- b Find the matrix of transformation that diagonalize the matrix $A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$. Also write the diagonal matrix. (7)

MODULE 2

- 13 a If $w = x^2 + y^2 - z^2$ where $x = \rho \sin \phi \cos \theta, y = \rho \sin \phi \sin \theta, z = \rho \cos \phi$. Find $\frac{\partial w}{\partial \rho}$ and $\frac{\partial w}{\partial \theta}$ using chain rule. (7)

- b Locate all relative extrema and saddle points of $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ (7)

- 14 a Show that the function $f(x, y) = 2 \tan^{-1}\left(\frac{y}{x}\right)$ satisfies the Laplace equation $f_{xx} + f_{yy} = 0$. (7)

- b Find the local linear approximation L of $f(x, y) = \ln(xy)$ at the point $P(1, 2)$. Compute the error in approximation f by L at the point $Q(1.01, 2.01)$. (7)

MODULE 3

- 15 a Evaluate $\iint_R (3x - 2y) dA$, where R is the region enclosed by the circle $x^2 + y^2 = 1$. (7)

- b Evaluate $\int_0^1 \int_{4x}^4 e^{-y^2} dy dx$ by reversing the order of integration. (7)

- 16 a Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} y(x^2 + y^2) dx dy$ using polar coordinates. (7)

- b Let G be the tetrahedron in the first octant bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, (a, b, c > 0)$ find the volume of G. (7)

MODULE 4

- 17 a Test the convergence of (i) $\sum_{k=1}^{\infty} \frac{4k^2 - 2k + 6}{8k^7 + k - 8}$ (ii) $\sum_{k=1}^{\infty} \left(\frac{k+1}{k}\right)^{k^2}$ (7)

- b Test the convergence of the series $1 + \frac{1.2}{1.3} + \frac{1.2.3}{1.3.5} + \frac{1.2.3.4}{1.3.5.7} + \dots$ (7)

- 18 a Show that the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+3}{k(k+1)}$ is conditionally convergent (7)

- b Test the convergence of (i) $\sum_{k=1}^{\infty} \frac{(k+3)!}{3! k! 3^k}$ (ii) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{2k-1}}$ (7)

MODULE 5

- 19 a Find the Fourier series expansion of $f(x) = x + x^2$ in the range $(-\pi, \pi)$. (7)

- b Obtain the half range Fourier sine series of $f(x) = \begin{cases} x, & 0 < x < 2 \\ 4 - x, & 2 < x < 4 \end{cases}$ (7)
- 20 a Find the Fourier series expansion of $f(x) = |x|$ in the range $(-\pi, \pi)$. Hence (7)
- show that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$
- b Obtain the half range Fourier cosine series of $f(x) = x^2$ in $0 < x < 2$ (7)
