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Reg No.:_______________ Name:__________________________

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**

Second Semester B.Tech Degree Examination July 2021

**Course Code: MAT102**

**Course Name: VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND TRANSFORMS**

**(2019 Scheme)**

**AN Session**

Max. Marks: 100 Duration: 3 Hours

## PART A

***Answer all questions, each carries 3 marks.***

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|---|---|---|
| 1 | Find the velocity and acceleration of a particle whose position vector is given by $\vec{r}(t) = e^{t}\,\vec{i} + e^{-t}\,\vec{j}$ at $t = 0$. | (3) |
| 2 | If C is the unit circle $x^2 + y^2 = 1$ oriented counter clockwise, find $\int_C x\,dx + y\,dy$. | (3) |
| 3 | Determine the sources and sinks of the vector field $\vec{f}(x,y,z) = 2\left(x^3 - 2x\right)\vec{i} + 2\left(y^3 - 2y\right)\vec{j} + 2\left(z^3 - 2z\right)\vec{k}$ . | (3) |
| 4 | Evaluate $\iint_\sigma x\,z\,dS$ where σ is the part of the plane $x + y + z = 1$ that lies in the first octant. | (3) |
| 5 | Solve the initial value problem $y'' + y' - 2y = 0,\ \ y(0) = 4,\ \ y'(0) = -5$ . | (3) |
| 6 | Solve $y'''' + 10y'' + 9y = 0$. | (3) |
| 7 | Find the Laplace transform of $e^{-3t}\cos 2t$ . | (3) |
| 8 | Find the inverse Laplace Transform of $\dfrac{1}{(s+a)(s+b)}$ | (3) |
| 9 | Find the Fourier cosine transform of the function $f(x) = \begin{cases} x & if\ \ 0 < x < a \\ 0 & if\ \ x > a \end{cases}$ | (3) |
| 10 | Using Fourier sine integral , show that $\displaystyle\int_0^\infty \frac{1 - \cos \pi w}{w} \sin x w\ dw = \begin{cases} \frac{\pi}{2} & if\ \ 0 < x < \pi \\ 0 & if\ \ x > \pi \end{cases}$ | (3) |

## PART B

***Answer one full question from each module, each question carries 14 marks***

### Module-I

11 a) Find the directional derivative of $\varphi(x,y,z) = x^3 z - y x^2 + z^2$ at (1,1,1) in the direction of $\vec{a} = 2\vec{i} - \vec{j} + 2\vec{k}$. Also find the maximum directional derivative. (7)

b) Show that the vector field $\vec{f}(x,y) = x y^2 \vec{i} + x^2 y \vec{j}$ is conservative and find $\phi$ such that $\vec{f} = \nabla\phi$. Hence evaluate $\int_{(1,2)}^{(2,4)} x y^2\, dx + x^2 y\, dy$ . (7)

12 a) Find the parametric equation of the tangent line to the graph $\vec{r}(t) = t^2\,\vec{i} - \frac{1}{t+3}\vec{j} + (4-t^2)\vec{k}$ at (4,-1,0). (7)

b) Using line integral evaluate $\int_C x^2 y\, dx + x\, dy$ where C is the triangular path connecting (0,0) ,(1,0) and (1,2) in the positive direction. (7)

### Module-II

13 a) Use Green's theorem to evaluate $\int_C \log(1+y)\, dx - \frac{xy}{1+y} dy$ where C is the triangle with vertices (0,0),(2,0) and (0,4). (7)

b) Use Divergence theorem to find the outward flux of the vector filed $\vec{F} = (2x + y^2)\vec{i} + x\,y\,\vec{j} + (xy - 2z)\vec{k}$ across the surface of the tetrahedron bounded by $x + y + z = 2$ and the coordinate planes. (7)

14 a) Find the flux of the vector field $\vec{F}(x,y,z) = x\vec{k}$ across the surface, the portion of the paraboloid $z = x^2 + y^2$ below the plane $z = 2y$ oriented by downward unit normal. (7)

b) Use Stokes theorem evaluate $\oint_C \vec{f}.dr$ where $\vec{f}(x,y,z) = (z-y)\vec{i} + (z+x)\vec{j} - (x+y)\vec{k}$ and C is the boundary of the paraboloid $z = 9 - x^2 - y^2$ above the XY-plane with upward orientation. (7)

### Module-III

15 a) Solve using the method of undetermined coefficients $y'' - 4y' - 5y = 4\cos 2x$. (7)

b) Solve using the Method of variation of parameters $y'' + y = \operatorname{cosec} x$ (7)

16 a) Solve using the method of undetermined coefficients (7)
$y'' - 7y' + 12y = e^{2x}, \quad y(0) = 1, \; y'(0) = 2$ .

b) Solve the boundary value problem $x^2 y'' - 3xy' + 4y = 0, \quad y(1) = 0, \; y'(1) = 1$ (7)

**Module-IV**

17 a) Using convolution theorem find the Laplace inverse of $\dfrac{2}{\left(s^2+1\right)\left(s^2+25\right)}$ (7)

b) Using Laplace Transform solve $y'' + 2y' + y = e^{-t} \quad y(0) = 0, \; y'(0) = 1$ (7)

18 a) Express in terms of unit step function and hence find the Laplace Transform of

$$f(t) = \begin{cases} t^2, & if \quad 0 < t < 2 \\ t - 1 & if \quad 2 < t < 3 \\ 7 & if \quad t > 3 \end{cases}$$ (7)

b) Find the inverse Laplace Transform of $\dfrac{s^2+2}{s\left(s^2+9\right)}$ (7)

**Module-V**

19 a) Find the Fourier Transform of $f(x) = \begin{cases} |x| & if \quad -1 < x < 1 \\ 0 & otherwise \end{cases}$ (7)

b) Find the Fourier integral of $f(x) = \begin{cases} \pi - x & if \quad 0 < x < \pi \\ 0 & otherwise \end{cases}$ (7)

20 a) Find the Fourier Sine Transform of $f(x) = \begin{cases} x & if \quad 0 < x < 1 \\ 3 - x & if \quad 1 < x < 3 \\ 0 & if \quad x > 3 \end{cases}$ (7)

b) Represent $f(x) = e^{-kx} \quad x > 0, \; k > 0$ as a Fourier Cosine integral (7)

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