Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Second Semester B.Tech Degree Examination July 2021 (2019 scheme)

Course Code: MAT102

Course Name: VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND TRANSFORMS

(2019 Scheme)

FN Session

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

1		Find the unit tangent vector at a point $t_0 = \frac{\pi}{6}$ to the curve $\bar{\boldsymbol{r}}(t) = \cos 3t\,\hat{\imath} + sin3t\,\hat{\jmath} + 3t\,\hat{k}$.	(3)
2		Find the directional derivative of the function $\varphi = 3x^2y - y^3z^2$ at $(-1,-2,-1)$in the direction of negative z axis.	(3)
3		Using Green's Theorem evaluate $\int_C 2xy\,dx + (x^2 + x)dy$ where C is the unit circle in the positive direction.	(3)
4		Determine whether the vector field $\vec{F}(x,y,z) = (y+z)\hat{\imath} - (xz^3)\hat{\jmath} + (x^2 siny)\hat{k}$ is free of sources and sinks.	(3)
5		Solve the initial value problem $y'' + y = 0: y(0) = 3, y'(0) = 1$.	(3)
6		Find the Wronskian corresponding to the solution of $y'' - 3y' + 2y = 0$.	(3)
7		Find the Laplace Transform of $sin3t\,cos2t$.	(3)
8		Evaluate $L^{-1}\left[\frac{2}{(s+4)^3}\right]$.	(3)
9		Find the Fourier cosine transform of the function $f(x) = \begin{cases} k, & 0 < x < a \\ 0, & x > a \end{cases}$	(3)
10		Express $f(x) = \begin{cases} \frac{1}{2}, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$ as a Fourier sine integral.	(3)

PART B

Answer one full question from each module, each question carries 14 marks

Module-I

11	a)	Use a line integral to evaluate the work done by the force field $\vec{F} = xy^2\hat{\imath} + xy\,\hat{\jmath}$ along the triangle with vertices $(0,0), (2,1)$ and $(0,1)$ in the positive direction.	(7)

b) Use the given information to find the position and velocity vectors of the particle with acceleration $\vec{a}(t) = -\cos t\,\hat{\imath} - \sin t\,\hat{\jmath}$, $\vec{v}(0) = \hat{\imath}$, $\vec{r}(0) = \hat{\jmath}$ (7)

12 a) Evaluate $\int_C (xyz)dx - \cos(yz)\,dy$ where C is the straight line segment from $(1,1,1)$ to $(-2,1,3)$. (7)

b) Show that the vector field $\vec{F}(x,y) = \cos y\,\hat{\imath} - x\sin y\,\hat{\jmath}$ is conservative and find φ such that $\vec{F} = \nabla\varphi$. Hence evaluate $\int_{(0,1)}^{(\pi,0)} \cos y\,dx - x\sin y\,dy$ (7)

Module-II

13 a) Using divergence theorem, evaluate $\iint_\sigma \vec{F}.\hat{n}\,dS$, where $\vec{F}(x,y,z) = (x^2+y)\,\hat{\imath} + z^2\hat{\jmath} + (e^y - z)\hat{k}$ and σ is the surface of rectangular cube bounded by the coordinate planes and the plane $x = 3, y = 1, z = 3$. (7)

b) Use Stoke's Theorem to evaluate the work done by the force field $\vec{F}(x,y,z) = 3z\,\hat{\imath} + 4x\,\hat{\jmath} + 2y\hat{k}$ over the boundary of the paraboloid $z = 4 - x^2 - y^2, z \geq 0$ with upward orientation. (7)

14 a) Let σ be the portion of the surface $z = 1 - x^2 - y^2$ that lies above the xy −plane and σ is the oriented upwards. Find the flux of the vector field $\vec{F}(x,y,z) = x\,\hat{\imath} + y\,\hat{\jmath} + z\hat{k}$ across σ. (7)

b) Find the mass of the lamina that is the portion of the plane $x + y + z = 2$ lying in the first octant where the density function of the surface is $\sigma = xz$. (7)

Module-III

15 a) Solve using the method of undetermined coefficients: $y'' - 4y' + 4y = 4\sin^2 x$. (7)

b) Solve using the method of variation of parameters: $y'' + 4y = \sec 2x$. (7)

16 a) Solve using the method of undetermined coefficients: $y''' + 2y'' - y' - 2y = e^x$. (7)

b) Solve the initial value problem $x^2y'' - 3xy' + 3y = 0, y(1) = 0, y'(1) = 1$. (7)

Module-IV

17 a) Using Laplace Transform solve $y'' + 4y' + 3y = e^{-t}, y(0) = 1, y'(0) = 1$. (7)

b) Using convolution theorem, find the inverse Laplace Transform of $\frac{18s}{(s^2+36)^2}$ (7)

18 a) Use Laplace Transform to solve $y'' + 3y' + 2y = u(t-1)$, $y(0) = 0, y'(0) = 0$ (7)

b) Evaluate $L^{-1}\left[\frac{2s+1}{s^2+2s+5}\right]$ (7)

Module-V

19 a) Find the Fourier integral representation of the function $f(x) = \begin{cases} 1, & if\ |x| < 1 \\ 0, & if\ |x| > 1 \end{cases}$. (7)

b) Find the Fourier transform of $f(x) = e^{-|x|}, -\infty < x < \infty$ (7)

20 a) Represent $f(x) = \begin{cases} \text{Sin}\, x, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$ as Fourier Cosine Integral. (7)

b) Find the Fourier sine transform of the function $f(x) = \begin{cases} 1, & 0 < x < 1 \\ -1, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$ (7)

****