Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Second Semester B.Tech Degree Examination June 2022 (2019 scheme)

Course Code: MAT102
Course Name: VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND TRANSFORMS
(2019-Scheme)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

1 Find the directional derivatives of $f(x,y) = x^2 - 3xy + y^2$ at the point P(2,1) in the direction of $\vec{a} = \frac{1}{3}\vec{i} + \frac{2}{3}\vec{j}$. (3)

2 Evaluate $\int_C 3xy\,dy$ where C is the line segment joining (0,0) and (1,2). (3)

3 Determine the sources and sinks of the vector field $\vec{f}(x,y) = x^2\vec{i} + y^2\vec{j} + z^2\,\vec{k}$. (3)

4 Using Divergence theorem evaluate $\iint_S \vec{f}.\vec{n}\,dS$ where $\vec{f} = 2x\vec{i} + 4y\,\vec{j} - 3z\,\vec{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ (3)

5 Solve the initial value problem $y'' + 5y' + 6y = 0,\ y(0) = 1,\ y'(0) = 2$ (3)

6 Solve $y''' - y' = 0$ (3)

7 Find the Laplace Transform of $(\sin t + \cos t)^2$ (3)

8 Find the inverse Laplace Transform of $\frac{e^{-3s}}{(s+2)^2}$ (3)

9 Find the Fourier sine transform of $e^{-x}\quad (x > 0)$ (3)

10 Find the Fourier Sine Integral of $f(x) = \begin{cases} \sin x & if \quad 0 < x < \pi \\ 0 & if \quad x > \pi \end{cases}$ (3)

PART B

Answer one full question from each module, each question carries 14 marks

Module-I

11 a) Find the parametric equation of the tangent line to the curve (7)

$\vec{r}(t) = 2\cos \pi t\, \vec{i} + 2\sin \pi t\, \vec{j} + 6t\, \vec{k}$ at the point $t = \frac{1}{3}$

b) Show that the vector field $\vec{f}(x,y) = 2xy^3\, \vec{i} + 3y^2x^2\, \vec{j}$ is conservative and find ϕ such that $\vec{f} = \nabla\phi$. Hence evaluate $\int_{(2,-2)}^{(-2,0)} 2xy^3\, dx + 3y^2x^2\, dy$. (7)

12 a) Find the position anrd velocity vectors of the particle given $\vec{a}(t) = (t+1)^{-2}\, \vec{j} - e^{-2t}\vec{k},\ \vec{v}(0) = 3\vec{i} - \vec{j},\ \vec{r}(0) = \vec{k}$ (7)

b) If $\vec{r} = x\vec{i} + y\, \vec{j} + z\, \vec{k}$ and let $\vec{F}(r) = f(r)\, \vec{r}$ prove that $div\vec{F} = 3f(r) + \vec{r}\, f'(r)$ (7)

Module-II

13 a) Use Green's theorem to find the work done by the force field $\vec{f}(x,y) = xy\vec{i} + \left(\frac{x^2}{2} + xy\right)\vec{j}$ on a particle that starts at (4,0) transverse the upper semicircle $x^2 + y^2 = 16$ and returns to its starting point along the x-axis. (7)

b) Find mass of the lamina that is a portion of cone $z = \sqrt{x^2 + y^2}$ that lies between the planes $z = 1$ and $z = 3$, if the density is $\phi(x,y,z) = x^2 z$ (7)

14 a) Let σ be the portion of the surface $z = 1 - x^2 - y^2$ that lies above the xy −plane and σ is the oriented upwards. Find the flux of the vector field $\vec{F}(x,y,z) = x\,\hat{\imath} + y\,\hat{\jmath} + z\hat{k}$ across σ. (7)

b) Use stokes theorem to evaluate $\oint_C \vec{F}.dr$ where $\vec{F}(x,y,z) = z^2\,\vec{\mathrm{i}} + 3x\,\vec{\mathrm{j}} - y^3\,\vec{\mathrm{k}}$ and C is the circle $x^2 + y^2 = 1$ in the XY plane with counter clockwise orientation looking down the positive Z axis (7)

Module-III

15 a) Using the method of undetermined coefficients solve, $y'' - 4y = xe^x$ (7)

b) Using the Method of variation of parameters solve, $y'' - 4y' + 5y = \frac{e^{2x}}{\sin x}$ (7)

16 a) Solve the initial value problem, by method of undetermined coefficients $y'' + 4y = 8x^2,\ y(0) = -3,\ y'(0) = 0$ (7)

b) Solve the initial value problem $x^2y'' + 3xy' + y = 0,\ y(1) = -3,\ y'(1) = 1$ (7)

Module-IV

17 a) Using Laplace Transform solve $y'' + 5y' + 6y = e^{-t} \quad y(0) = 0,\ y'(0) = 1$ (7)

b) Using convolution theorem find the Inverse Laplace Transform of $\dfrac{s^2}{\left(s^2 + a^2\right)\left(s^2 + b^2\right)}$ (7)

18 a) Find the inverse Laplace Transform of $\dfrac{s+8}{s^2 + 4s + 5}$ (7)

b) Using Laplace Transform solve $y'' + 16y = 4\delta(t - 3\pi) \quad y(0) = 2,\ y'(0) = 0$ (7)

Module-V

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a) Find the Fourier Transform of $f(x) = \begin{cases} e^x & if \quad -a < x < a \\ 0 & otherwise \end{cases}$ (7)

b) Find the Fourier Cosine integral of $f(x) = \begin{cases} \cos x & if \quad 0 < x < \dfrac{\pi}{2} \\ 0 & otherwise \end{cases}$ (7)

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a) Find the Fourier Cosine Transform of $f(x) = \begin{cases} x^2 & if \quad 0 < x < 1 \\ 0 & if \quad x > 1 \end{cases}$ (7)

b) Find the Fourier Transform of $f(x) = \begin{cases} a - |x| & if \quad |x| < a \\ 0 & otherwise \end{cases}$ (7)

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