Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Second Semester B.Tech Degree Regular and Supplementary Examination June 2023 (2019 Scheme)

Course Code: MAT 102

Course Name: VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND TRANSFORMS

(2019 SCHEME)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all Questions. Each question carries 3 Marks Marks

1	Find the parametric equation of the tangent vector of the curve $\overrightarrow{r(t)} = t^2\,\hat{\imath} + 2t^3\,\hat{\jmath} + 3t\,\hat{k}\ at\ t = 1.$	(3)
2	Find the directional derivative of $f(x,y) = xe^y$ at (1,1) in the direction of the vector $\hat{\imath} - \hat{\jmath}$.	(3)
3	Use Green's theorem to evaluate $\oint_C\ xdy - ydx$, where C is the circle $x^2 + y^2 = 4$.	(3)
4	Determine whether the vector field $\vec{F} = 4(x^3 - x)\hat{\imath} + 4(y^3 - y)\hat{\jmath} + 4(z^3 - z)\hat{k}$ is free of sources and sinks. If not locate them.	(3)
5	Show that the functions $x, x\ lnx$ are linearly independent.	(3)
6	Solve the differential equation $y'' + 4y' + 2.5y = 0$.	(3)
7	Find the Laplace transform of $\sin 4t \cos 3t$.	(3)
8	Find the Laplace transform of $e^{-3t}u(t-1)$.	(3)
9	Determine the Fourier sine Transform of $f(x) = 3x,\quad 0 < x < 6$.	(3)
10	Find the Fourier sine integral of $f(x) = \begin{cases} sinx & if\ 0 < x < \pi \\ 0 & if\ x > \pi \end{cases}$	(3)

PART B

Answer one full question from each module, each question carries 14 marks

Module I

11	a	Find the divergence and curl of the vector field $\vec{F}(x,y,z) = zy\hat{\imath} + y^2x\hat{\jmath} + yz^2\hat{k}$.	(7)
	b	Show that $\vec{F} = (cosy + ycosx)\hat{\imath} + (sinx - xsiny)\hat{\jmath}$ is a conservative vector field. Hence find a potential function for it.	(7)

OR

12	a	Find the work done by the force $\vec{F} = xy\hat{\imath} + yz\hat{\jmath} + xz\hat{k}$ on a particle that moves along the curve $C: \vec{r}(t) = t\hat{\imath} + t^2\hat{\jmath} + t^3\hat{k}$ where $0 \le t \le 1$.	(7)
	b	Show that $\int_c (3x^2e^y dx + x^3e^y dy)$ is independent of the path and hence evaluate the integral from (0,0) to (3,2).	(7)

Module II

13	a	Using Green's theorem, evaluate the line integral $\int_c (xy + y^2)\, dx + x^2 dy$ where C is bounded by $y = x \; and \; y = x^2$ and positively oriented.	(7)
	b	Evaluate the surface integral $\iint_\sigma z^2\, dS$, where σ is the portion of the cone $z = \sqrt{x^2 + y^2}$ between the planes z=1 and z=3.	(7)

OR

14	a	Evaluate $\iint_s \vec{F}.\hat{n}dS$, where S is the surface of the cylinder $x^2 + y^2 = 4, z = 0, z = 3$ where $\vec{F} = (2x - y)\vec{\imath} + (2y - z)\vec{\jmath} + z^2\vec{k}$.	(7)
	b	Apply Stoke's theorem to evaluate $\int_C (x - y)dx + (y - z)dy + (z - x)dz$ where C is the boundary of the portion of the plane $x + y + z = 1$ in the first octant.	(7)

Module III

15	a	Solve the Cauchy -Euler differential equation $(x^2D^2 - 3xD + 10)y = 0$.	(7)
	b	Solve the initial value problem $y'' - 2y' + 5y = 0, y(0) = -3, y'(0) = 1$.	(7)

OR

16	a	By the method of undetermined coefficients, solve $y'' + y' - 2y = sinx$.	(7)
	b	Using method of variation of parameters solve $\frac{d^2y}{dx^2} + 4y = sec2x$.	(7)

Module IV

17	a	Find the Laplace transform of $\cos^2 t$.	(7)
	b	Find the inverse Laplace transform of $\frac{3s+2}{(s-1)(s^2+1)}$.	(7)

OR

18	a	Using Laplace transform solve $y'' + 5y' + 6y = e^{-2t}$ given that $y(0) = y'(0) = 1$	(7)
	b	Find the inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$ using convolution.	(7)

Module V

19 a Find the Fourier transform and integral representation of $f(x) = \begin{cases} 1, & |x| < 1 \\ o, & otherwise \end{cases}$. Hence show that $\int_0^\infty \frac{sinw}{w}\ dw = \pi/2$ (7)

b Find the Fourier sine transform and inverse transform of $e^{-ax}, x > 0\ \ a > 0$ (7)

OR

20 a Find the complex Fourier transform of $f(x) = \begin{cases} sinx, & |x| \le a, a > 0 \\ 0\ , & |x| > a \end{cases}$ (7)

b Find the Fourier transform and integral representation of (7)

$$f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0\ , & otherwise \end{cases}$$

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