Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Second Semester B.Tech Degree (R, S) Examination May 2024 (2019 Scheme)

Course Code: MAT 102

Course Name: VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND TRANSFORMS

(2019 SCHEME)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all Questions. Each question carries 3 Marks Marks

1	If $\bar{r} = e^{-t}\,\hat{\imath} + e^{t}\,\hat{\jmath}$ is the position vector of a moving particle, find its velocity at $t = 0$.	(3)
2	Find a unit vector in the direction in which $f(x,y) = 4x^3y$ increases most rapidly at P(-1, 1), and find the rate of change of f at P in that direction.	(3)
3	Evaluate $\int_C (x^2 - 3y)dx + 3x\,dy$, using Green's theorem, C being the circle $x^2 + y^2 = 4$.	(3)
4	Determine whether the vector field $\bar{F}(x,y,z) = x^3\,\hat{\imath} + y^3\,\hat{\jmath} + z^3\hat{k}$ is free of sources and sinks. If it is not, locate them.	(3)
5	Find whether the solution set $\{x\,sinx, x\,cosx\}$ forms a basis or not.	(3)
6	Solve $y''' + y'' = 0$.	(3)
7	Find the Laplace Transform of $e^{-2t}\,sin5t$.	(3)
8	Find the inverse Laplace Transform of $\frac{4}{(s+1)^4}$	(3)
9	Find the Fourier cosine integral of $f(x) = \begin{cases} 1, & if\ \lvert x\rvert < 1, \\ 0, & if\ \lvert x\rvert > 1 \end{cases}$	(3)
10	Find the Fourier sine transform of e^{-x}.	(3)

PART B

Answer one full question from each module, each question carries 14 marks

Module I

11	a	If $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ $and\ r = \lvert\bar{r}\rvert$, prove that $\nabla f(r) = \frac{f'(r)}{r}\bar{r}$.	(7)
	b	Prove that the line integral $\int_{(-1,2)}^{(0,1)} (3x - y + 1)dx - (x + 6y + 2)dy$ is independent of the path. Also find its value.	(7)

OR

12	a	Find the work done by the force field $\bar{F}(x,y,z) = z\,\hat{\imath} + x\,\hat{\jmath} + y\,\hat{k}$, where C is the curve $\bar{r}(t) = sint\,\hat{\imath} + 4sint\,\hat{\jmath} + sin^2t\,\hat{k}, 0 \le t \le {}^{\pi}/_{2}$.	(7)
	b	Find $\nabla \times (\nabla \times \bar{f})$, if $\bar{f} = y^2x\,\hat{\imath} - 3yz\,\hat{\jmath} + xz\,\hat{k}$	(7)

Module II

13	a	Find the area of the ellipse $x = a\,cost,\ y = b\,sint;\ 0 \le t \le 2\pi$ using line integrals.	(7)
	b	Use Stoke's theorem to evaluate $\int_c \bar{F}\,.\,d\bar{r}$, where $\bar{F}(x,y,z) = xy\,\hat{\imath} + x^2\,\hat{\jmath} + z^2\,\hat{k}$ and C is the intersection of the rectangle $0 \le x \le 1, 0 \le y \le 3,$ and the plane $z = y$.	(7)

OR

14	a	Using Green's theorem evaluate $\int_c x\,cosy\,dx - y\,sinx\,dy$ where C is the square with vertices $(0,0), (0,\ \pi), (\pi,\ \pi)$, and $(\pi,\ 0)$.	(7)
	b	If $\bar{F} = x^3\hat{\imath} + y^3\hat{\jmath} + z^3\hat{k}$, σ is the surface of the cylinder bounded by $x^2 + y^2 = 4,\ z = 0,\ z = 4$, find the outward flux of $\bar{F}$ across σ using Divergence theorem.	(7)

Module III

15	a	Solve the initial value problem $(D^2 + 4D + 5)y = 0,\ \ y(0) = 2,\ y'(0) = -5$.	(7)
	b	Solve $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$, by the method of variation of parameters.	(7)

OR

16	a	Solve by the method of undetermined coefficients, $y'' - 4y' + 3y = sin\,3x$.	(7)
	b	Solve $x^2y'' + 7xy' + 13y = 0,\ \ y(1) = 0,\ y'(1) = 2$.	(7)

Module IV

17	a	Solve the differential equation using Laplace transform, $y'' + 2y' + 6y = 6te^{-t}$, given that y(0)=2, $y'(0) = 5$.	(7)
	b	Find the inverse Laplace transform of (i) $\frac{2s+1}{s^2+2s+5}$ (ii) $\frac{e^{-s}}{s^2+2s+1}$.	(7)

OR

18	a	Using convolution find the inverse Laplace transform of $\frac{1}{s^2(s^2+a^2)}$.	(7)
	b	Find the Laplace transform of (i) $sin^2\,3t$ (ii) $t^2\ \ in\ 1 \le t \le 2$.	(7)

Module V

19 a Using Fourier integrals prove that (7)

$$\int_0^\infty \frac{\cos\left(\frac{\pi\omega}{2}\right)}{1-\omega^2} \cos\omega x \, d\omega = \begin{cases} \frac{\pi}{2}\cos x, & |x| < \pi/2 \\ 0, & |x| > \pi/2 \end{cases}$$

b Find the Fourier Cosine transform of $f(x) = \begin{cases} x, & if\ 0 < x < 2, \\ 0, & if\ x > 2 \end{cases}$ (7)

OR

20 a Find the Fourier sine integrals of $f(x) = \begin{cases} \pi - x, & if\ 0 < x < \pi, \\ 0, & if\ x > \pi \end{cases}$ (7)

b Find the Fourier sine transform of $f(x) = \begin{cases} 1, & if\ 0 < x < 1, \\ 0, & if\ x > 1 \end{cases}$ (7)

Hence deduce that $\int_0^\infty \frac{1-\cos\omega}{\omega} sin\left(\frac{\omega}{2}\right) d\omega = \begin{cases} \frac{\pi}{2}, & if\ 0 < x < 1, \\ 0, & if\ x > 1 \end{cases}$

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