Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

First Semester B.Tech Degree (S, FE) Examination June 2024 (2019 Scheme)

Course Code: MAT 101

Course Name: LINEAR ALGEBRA AND CALCULUS

(2019 -Scheme)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks Marks

1	Find the rank of the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$.	(3)
2	Find the sum and product of eigen values of $A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 2 & 6 \\ 0 & 0 & 6 \end{bmatrix}$ without finding the characteristic equation.	(3)
3	Find the slope of the sphere $x^2 + y^2 + z^2 = 14$ in the y direction at $(1,2,3)$	(3)
4	Show that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$, where $z = 10x^5y^3 + 5x + 2y$	(3)
5	Find the area of the region bounded by $y = x^2$ and $y = x$.	(3)
6	Evaluate $\int_2^4 \int_1^3 (40 - 2xy)\,dxdy$.	(3)
7	Test the convergence of the series $\sum_{k=1}^{\infty} \frac{99^k}{k!}$	(3)
8	Test the convergence of the series $\sum_{k=1}^{\infty} \frac{k}{k+1}$	(3)
9	Find the Maclaurin series for the function $f(x) = xe^x$	(3)
10	Write Binomial series for $(1 + x^2)^3$	(3)

PART B

Answer one full question from each module, each question carries 14 marks.

MODULE 1

11 a Solve the following system of equations using Gauss elimination method (7)

$y - 3z = -1$

$x + z = 1$

$3x + y = 2$

$x + y - 2z = 0$

	b	Find the eigen values and eigen vectors of the matrix A=$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.	(7)
12	a	Find the matrix of the transformation that diagonalise the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. Also write the diagonal matrix.	(7)
	b	Find the value of α for which the system of equation is consistent. $x + y + z = 1$ $x + 2y + 3z = \alpha$ $x + 5y + 9z = \alpha^2$	(7)

MODULE 2

13	a	Find the local linear approximation L of $f(x, y, z) = \log(x + yz)$ at the point $(2,1,-1)$.	(7)
	b	If $w = f(P, Q, R)$ where $P = 2x - 3y,\ Q = 3y - 4z,\ R = 4z - 2x$, then prove that $\frac{1}{2}\frac{\partial w}{\partial x} + \frac{1}{3}\frac{\partial w}{\partial y} + \frac{1}{4}\frac{\partial w}{\partial z} = 0$	(7)
14	a	Locate all relative extrema and saddle points of $x^3 + y^3 - 3xy = 0$.	(7)
	b	Find the differential dw of the functions. i) $w = \frac{xyz}{x+y+z}$ ii) $w = e^{xy}$	(7)

MODULE 3

15	a	Evaluate $\iint_R \frac{1}{1+x^2+y^2}\, dA$ where R is the sector in the first quadrant bounded by $y = 0,\ y = x,\ x^2 + y^2 = 9$.	(7)
	b	Evaluate the integral $\int_0^4 \int_y^4 \frac{x}{x^2+y^2}\, dxdy$ by reversing the order of integration.	(7)
16	a	Use triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$.	(7)
	b	Find the center of gravity of a triangular lamina with vertices (0,0), (0,1) and (1,0) and density function $\rho(x, y) = xy$ and mass $= \frac{1}{24}$.	(7)

MODULE 4

17	a	A ball is dropped from a height of 10m. Each time it strikes the ground it	(7)

bounces vertically to a height that is $\frac{2}{3}$ of the preceding height. Find the total distance travelled by the ball, if it is assumed to bounce infinitely often.

b Check the convergence the following series (7)

i) $\sum_{n=1}^{\infty} \frac{n}{(2n-1)(2n+1)(2n+3)}$

ii) $\sum_{n=1}^{\infty} \left(\frac{n}{n^2+1}\right)^{n^2}$

18 a Show that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$ is conditionally convergent. (7)

b Check the convergence of the series $\frac{1}{3} + \frac{1.2}{3.5} + \frac{1.2.3}{3.5.7} + \cdots$ (7)

MODULE 5

19 a Find the Taylor series expansion of $f(x) = x sinx$ about the point $x = \frac{\pi}{2}$ (7)

b Find the Fourier series representation of $f(x) = x^2$ in $[-\pi, \pi]$ and deduce that $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$ (7)

20 a Find the half range Fourier cosine series of $f(x) = cosx$ in $0 \le x \le \frac{\pi}{2}$ (7)

b Find the half range Fourier sine series of $f(x) = e^x$ in $(0,1)$ (7)

****